SISTEMAS DIGITAIS – FOLHA 3

ÁLGEBRA DE BOOLE . SIMPLIFICAÇÃO DE FUNÇÕES LÓGICAS
MIN TERMOS (POS) E MAX TERMOS (SOP)

---

1. Mostre que

(a) $(\bar{A}+\bar{B})(A+B) = \bar{A}B + A\bar{B}$

(b) $(AB+C)B = AB\bar{C} + \bar{A}BC + ABC$

(c) $BC + AD = (B+A)(B+D)(A+C)(C+D)$

(d) $(AB+AC+BC)\cdot\overline{ABC} = AB\bar{C} + AC\bar{B} + BC\bar{A}$

2. Simplifique as expressões

(a) $f_1 = A\bar{C} + B\bar{C}D + A\bar{B}C + ACD$

(b) $f_2 = B + \bar{A}\bar{B} + ACD + A\bar{C}$

(c) $f_3 = B\bar{C}D + \bar{A}B\bar{D} + AB\bar{C} + A\bar{B}D + A\bar{C}D$

3. Encontre o complemento $(\bar{f})$ das seguintes expressões (Leis de De Morgan):

(a) $f_1 = A + \bar{B}C$

(b) $f_2 = A(B+C) + B\bar{D}(\bar{A}+C)$

4. Implemente as funções seguintes usando portas AND, OR, e INVERT (E, OU, E INVERSOR)

(a) $f_1 = \bar{A} + B(\bar{C}+\bar{D})$

(b) $f_2 = (\bar{A}+B)(\bar{B}+C) + (AB+C)$

5. Construa (i) a tabela de verdade para as funções seguintes:

(a) $f_1 = A + B\bar{C}$

(b) $f_2 = AC + BC + AB$

(c) $f_3 = (A + \bar{B})(\bar{A} + \bar{C} + D)$

e (ii) obtenha as formas canónicas SOP ("Sum of Products") e POS ("Product of Sums")

SISTEMAS DIGITAIS – SOLUÇÃO FOLHA 3

SIMPLIFICAÇÃO DE FUNÇÕES LÓGICAS.

---

(POSTULADOS)

– A variável lógica A toma apenas dois valores $A \in \{V, F\}$, $A \in \{H, L\}$, $A \in \{0, 1\}$, etc.

– Existem 3 funções lógicas elementares

AND (E): $f = A \cdot B$


Tabela de Verdade

| A | B | $f = A \cdot B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

OR (OU): $f = A + B$

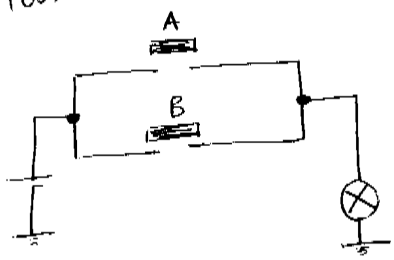



Tabela de verdade

| A | B | $f = A + B$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

NOT (NÃO): $f = \overline{A}$

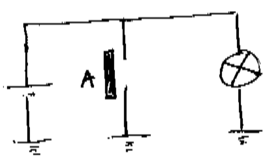



Tabela de Verdade

| A | $f = \overline{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

(leis)

Comutativa $A+B = B+A$

$A \cdot B = B\ A$

Associativa $(A+B)+C = A+(B+C)$

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Distributiva $A+(B \cdot C) = (A+B) \cdot (A+C)$

$A \cdot (B+C) = (A \cdot B)+(A C)$

Elemento Neutro $A+0 = A$

$A \cdot 1 = A$

Absorpção $A+1 = 1$

$A \cdot 0 = 0$

Complemento $A+\bar{A} = 1$

$A \cdot \bar{A} = 0$

Idempotência $A \cdot A = A$

$A+A = A$

De Morgan $\overline{A+B} = \bar{A} \cdot \bar{B}$ $(\sim(A \vee B) = \sim A \wedge \sim B)$

$\overline{A \cdot B} = \bar{A}+\bar{B}$ $(\sim(A \wedge B) = \sim A \vee \sim B)$

---

1a) (prop distributiva)

$(\bar{A}+\bar{B}) \cdot (A+B) = (\bar{A}+\bar{B}) \cdot A + (\bar{A}+\bar{B}) \cdot B$

$= \bar{A} A + \bar{B} A + \bar{A} B + \bar{B} B$

$= 0 + \bar{B} A + \bar{A} B + 0$

$= \bar{A} B + A \bar{B}$

1b) $(AB+C)B = ABB + CB$

$= AB + CB$

$= AB(C+\bar{C}) + (A+\bar{A})CB$

$= ABC + AB\bar{C} + ACB + \bar{A}CB$

$= AB\bar{C} + \bar{A}CB + ABC$

1c) $BC + AD \overset{\text{(prop distributiva)}}{=} (BC+A)\cdot(BC+D)$

$= (A+B)(A+C)\cdot(D+B)(D+C)$

$= (B+A)(B+D)(A+C)(C+D)$

1d) $(AB+AC+BC)\,\overline{ABC} \overset{\text{De Morgan}}{=} (AB+AC+BC)(\bar{A}+\bar{B}+\bar{C})$

$= (AB+AC+BC)\bar{A} + (AB+AC+BC)\bar{B} + (AB+AC+BC)\bar{C}$

$= \underbrace{\cancel{A\bar{A}B}}_{=0} + \underbrace{\cancel{A\bar{A}C}}_{=0} + \bar{A}BC + \underbrace{\cancel{AB\bar{B}}}_{=0} + A\bar{B}C + \underbrace{\cancel{B\bar{B}C}}_{=0} + AB\bar{C} + \underbrace{\cancel{AC\bar{C}}}_{=0} + \underbrace{\cancel{BC\bar{C}}}_{=0}$

$= AB\bar{C} + \bar{A}BC + A\bar{B}C = AB\bar{C} + AC\bar{B} + BC\bar{A}$

---

2a) $f_1 = \underbrace{A\bar{C}}_{=} + B\bar{C}D + A\bar{B}C + ACD =$

$= A\bar{C} + A\bar{C}\bar{B} + A\bar{C}D + B\bar{C}D + A\bar{B}C + ACD =$

$= A\bar{C} + B\bar{C}D + A\bar{B}(\bar{C}+C) + AD(\bar{C}+C)$

$= A\bar{C} + B\bar{C}D + A\bar{B} + AD$

[redacted]

2b) $f_2 = B + \bar{A}\bar{B} + ACD + A\bar{C}$

$$= \underbrace{(B+\bar{A})\cdot(B+\bar{B})}_{=1} + ACD + A\bar{C} + A\bar{C}D$$

$$= B + \bar{A} + A\bar{C} + AD(C+\bar{C})$$

$$= B + \bar{A} + A\bar{C} + AD$$

$$= B + \underbrace{\bar{A} + A\bar{C}}_{\text{prop distr}} + \underbrace{\bar{A} + AD}_{\text{prop distr}}$$

$$= B + \underbrace{(\bar{A}+A)}_{=1}(\bar{A}+\bar{C}) + \underbrace{(\bar{A}+A)}_{=1}(\bar{A}+D)$$

$$= B + \bar{A} + \bar{C} + \bar{A} + D$$

$$= B + \bar{A} + \bar{C} + D$$

2c) $f_3 = B\bar{C}D + \bar{A}B\bar{D} + AB\bar{C} + A\bar{B}D + A\bar{C}D$

$$= (A+\bar{A})B\bar{C}D + \bar{A}B(C+\bar{C})\bar{D} + AB\bar{C}(D+\bar{D}) + A\bar{B}(C+\bar{C})D$$

$$+ A(B+\bar{B})\bar{C}D$$

$$= \underset{1}{AB\bar{C}D} + \underset{2}{\bar{A}B\bar{C}D} + \underset{4}{\bar{A}BC\bar{D}} + \underset{4\ 2}{\bar{A}B\bar{C}\bar{D}} + \underset{rep}{\cancel{AB\bar{C}D}} + \underset{1}{AB\bar{C}\bar{D}} + \underset{3}{A\bar{B}CD}$$

$$+ \underset{3}{A\bar{B}\bar{C}D} + \underset{rep}{\cancel{AB\bar{C}D}} + \underset{rep}{\cancel{A\bar{B}\bar{C}D}}$$

$$= \overset{1}{AB\bar{C}(D+\bar{D})} + \overset{2}{\bar{A}B\bar{C}(D+\bar{D})} + \overset{4}{\bar{A}B(C+\bar{C})\bar{D}} + \overset{3}{A\bar{B}(C+\bar{C})D}$$

$$= AB\bar{C} + \bar{A}B\bar{C} + \bar{A}B\bar{D} + A\bar{B}D$$

$$= B\bar{C} + \bar{A}B\bar{D} + A\bar{B}D$$

3a) $\bar{f_1} = \overline{A + \bar{B}C}$

$$= \bar{A} \cdot \overline{\bar{B}C} = \bar{A} \cdot (B + \bar{C})$$

3b) $\bar{f_2} = \overline{A(B+C) + B\bar{D}(\bar{A}+C)}$

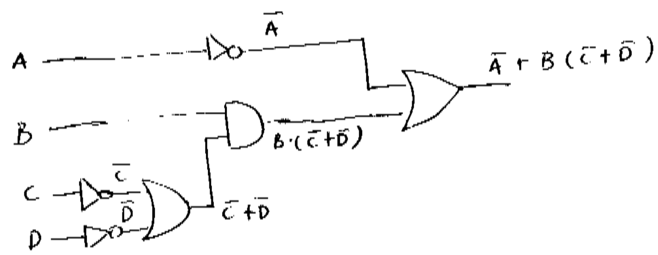
$$= \overline{A(B+C)} \cdot \overline{B\bar{D}(\bar{A}+C)}$$

$$= \left[\bar{A} + \overline{(B+C)}\right]\left[\bar{B} + D + \overline{(\bar{A}+C)}\right]$$

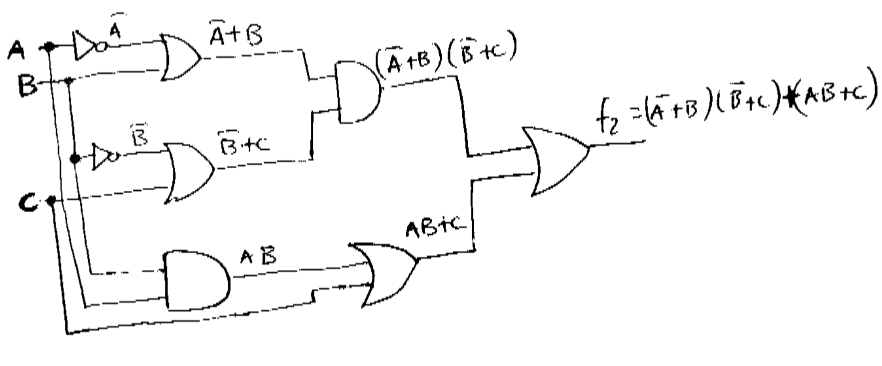
$$= (\bar{A} + \bar{B}\bar{C}) \cdot (\bar{B} + D + A\bar{C})$$

---

4a) $f_1 = \bar{A} + B(\bar{C} + \bar{D})$




4b) $f_2 = (\bar{A} + B)\cdot(\bar{B} + C) + (AB + C)$



5a) $f_1 = A + B\bar{C}$

| m | A | B | C | $\bar{C}$ | $B\bar{C}$ | $f = A + B\bar{C}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1 | 1 | 1 |
| 3 | 0 | 1 | 1 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 | 1 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 | 0 | 1 |
| 6 | 1 | 1 | 0 | 1 | 1 | 1 |
| 7 | 1 | 1 | 1 | 0 | 0 | 1 |

A função é verdadeira (igual a 1) quando

$f = \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$ ← min termos

$= \sum(2,4,5,6,7)$ ← forma canónica SOP (Soma de Produtos)

A função é falsa quando (igual a zero)

$\bar{f} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC$

$= \sum(0,1,3)$

Aplicando as leis de De Morgan

$\bar{\bar{f}} = f = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC}$

$= (\overline{\bar{A}\bar{B}\bar{C}}) \cdot (\overline{\bar{A}\bar{B}C}) \cdot (\overline{\bar{A}BC})$

$= (A+B+C) \cdot (A+B+\bar{C})(A+\bar{B}+\bar{C})$ ← Max termos

$= \prod(7,6,4)$ ← forma canónica POS (Produto de Somas)

$= \prod(7-0, 7-1, 7-3)$

5b) $f_2 = AC + BC + AB$

| m | A | B | C | AC | BC | AB | f = AC + BC + AB |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$$f = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$= \sum(3,5,6,7) \leftarrow \text{SOP}$$

$$\bar{f} = \sum(0,1,2,4)$$

$$\bar{f} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$\bar{\bar{f}} = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}}$$

$$f = \overline{\bar{A}\bar{B}\bar{C}} \cdot \overline{\bar{A}\bar{B}C} \cdot \overline{\bar{A}B\bar{C}} \cdot \overline{A\bar{B}\bar{C}}$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)$$

$$= \prod(7,6,5,3) \leftarrow \text{POS}$$

$$= \prod(7-0, 7-1, 7-2, 7-4)$$

5c)

$$f = (A + \bar{B}) \cdot (\bar{A} + \bar{B} + D)$$

| m | A | B | C | D | $\bar{A}$ | $\bar{B}$ | $A+\bar{B}$ | $\bar{A}+\bar{B}+D$ | f |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 10 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 11 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 12 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 14 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |

$$f = \sum (0,1,2,3,8,9,10,11,13,15)$$

$$\bar{f} = \sum (4,5,6,7,12,14)$$

Logo

$$f = \prod (15-4,\ 15-5,\ 15-6,\ 15-7,\ 15-12,\ 15-14)$$

$$= \prod (11,\ 10,\ 9,\ 8,\ 3,\ 1)$$