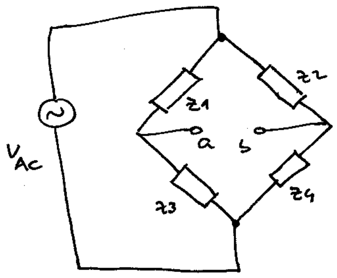
INSTRUMENTAÇÃO

AULA 2 — PONTES AC

ponte genérica



Zs são impedâncias

ponte em equilíbrio:

$V_{ab} = 0 \Rightarrow I_{ab} = 0$

$$V_a = \frac{Z_3}{Z_1 + Z_3} V_{AC}$$

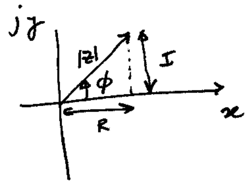
$$V_b = \frac{Z_4}{Z_2 + Z_4} V_{AC}$$

$$V_{ab} = V_a - V_b = \frac{Z_3(Z_2+Z_4) - Z_4(Z_1+Z_3)}{(Z_1+Z_3)(Z_2+Z_4)}$$

$$= Z_3 Z_2 + \cancel{Z_3 Z_4} - Z_4 Z_1 - \cancel{Z_4 Z_3} = 0$$

$\boxed{Z_3 Z_2 = Z_1 Z_4}$ ponte em equilíbrio

notação fasorial



$Z = R + jI$

$Z = |Z| e^{j\phi}$

com $|Z| = \sqrt{R^2 + I^2}$

$\phi = \operatorname{arctg}\left(\frac{I}{R}\right)$

$$Z_3 Z_2 = Z_1 Z_4$$

$$|Z_3| e^{j\phi_3} |Z_2| e^{j\phi_2} = |Z_1| e^{j\phi_1} |Z_4| e^{j\phi_4}$$

$$\boxed{\begin{array}{c} |Z_3| \, |Z_2| = |Z_1| \, |Z_4| \\ \phi_3 + \phi_4 = \phi_2 + \phi_1 \end{array}}$$

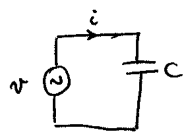
produto das magnitudes igual

soma das fases igual

notaçaõ fasorial:

$$z = |z|\, e^{j\phi} \longrightarrow z = |z| \angle\phi$$

impedância de um condensador



$$v = V e^{j\omega t}$$

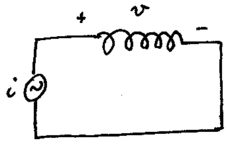
$$i = C \frac{dv}{dt} \quad \text{no condensador}$$

$$i = C V j\omega\, e^{j\omega t}$$

$$i = j\omega C v$$

$$\boxed{Z_C = \frac{v}{i} = \frac{1}{j\omega C}}$$

impedância de uma bobina



$$i = I e^{j\omega t}$$

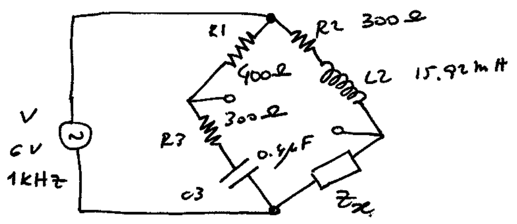
$$v = L \frac{di}{dt} \quad \text{na bobina}$$

$$v = L j\omega I e^{j\omega t}$$

$$v = j\omega L i$$

$$\boxed{Z_L = \frac{v}{i} = j\omega L}$$

Exemplo



Ponte está em equilíbrio.
Qual o valor de Z_x?

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_1 = R_1 = 400\,\Omega$$

$$Z_2 = R_2 + Z_{L2} = R_2 + j\omega L_2$$

$$= 200\,\Omega + j\, 2\pi \cdot 1\,KHz \cdot 15.92\ mH$$

$$= 200\,\Omega + j\,100\,\Omega = 223.6 \angle 26^\circ$$

$$Z_3 = R_3 + Z_{C3} = R_3 - \frac{j}{\omega C_3} = 300\,\Omega - j\frac{1}{2\pi \cdot 1\,KHz \cdot 0.4\,\mu F}$$

$$= 300\,\Omega - j\,400\,\Omega$$

$$= 500 \angle -53^\circ$$

$$Z_x = \frac{Z_2 Z_3}{Z_1}$$

$$Z_x = \frac{223.6 \angle 26^\circ \cdot 500 \angle -53^\circ}{400 \angle 0^\circ}$$

$$Z_x = 279.5\,\Omega \angle -26^\circ$$

$$Z_x = 279.5\, e^{-j\,26^\circ} = 279.5 \cos(26^\circ) - j\, 279.5 \sin(26^\circ)$$

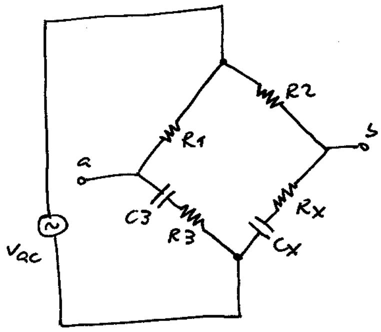
$$= 250.35 - j\, 124.28$$

Logo

$$R_4 = 250.35\,\Omega$$

$$Z_{C4} = -\frac{j}{\omega C_4} = -j\,124.28 \longrightarrow C_4 = \frac{1}{\omega \cdot 124.28} = 1.28\,\mu F$$

Ponte com braços similares (similar angle bridge)



R_x, C_x valores desconhecidos

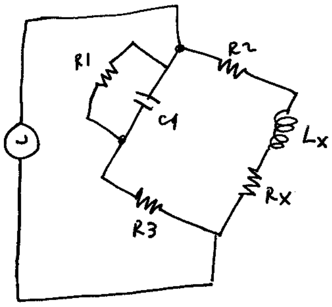
em equilíbrio:

$$R_1\left(R_x + \frac{1}{j\omega C_x}\right) = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$$

$$R_1 R_x - j\frac{R_1}{\omega C_x} = R_2 R_3 - j\frac{R_2}{\omega C_3}$$

$$\boxed{R_x = \frac{R_3 R_2}{R_1} \qquad C_x = \frac{R_1}{R_2} C_3}$$

Ponte de Maxwell



R_x, L_x desconhecidos

em equilíbrio

$$\frac{R_1 Z_{C1}}{R_1 + Z_{C1}} \cdot (R_x + Z_{Lx}) = R_2 R_3$$

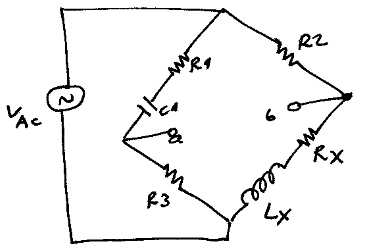
$$\frac{\frac{R_1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}\left(R_x + j\omega L_x\right) = R_2 R_3$$

$$\frac{R_1}{1 + j\omega R_1 C_1}\left(R_x + j\omega L_x\right) = R_2 R_3$$

$$R_1 R_x + j\omega R_1 L_x = R_2 R_3 + j\omega R_1 R_2 R_3 C_1$$

$$\boxed{R_x = \frac{R_2 R_3}{R_1} \quad ; \quad L_x = R_2 R_3 C_1}$$

Ponte de ângulos opostos



R_x, L_x desconhecidos

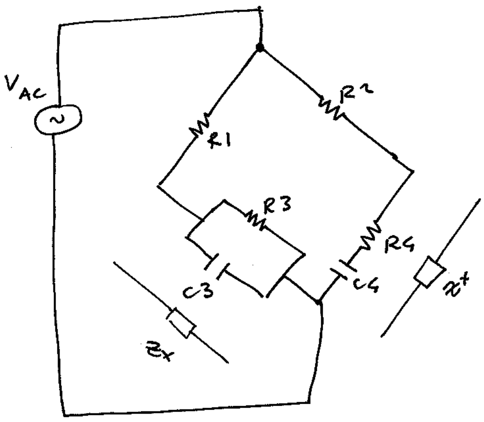
em equilíbrio:

$$R_x = \frac{\omega^2 R_1 R_2 R_3 C_1^2}{1 + \omega^2 R_1^2 C_1^2}$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 R_1^2 C_1^2}$$

NOTAR: dependência de ω

Ponte de WIEN



Serve para medir a impedância equivalente série ou equivalente paralelo de uma impedância desconhecida

equiv. paralelo:

$$R_3 = \frac{R_1}{R_2}\left(R_4 + \frac{1}{\omega^2 R_4 C_4^2}\right)$$

$$C_3 = \frac{R_2}{R_1} \frac{1}{1+\omega^2 R_4^2 C_4^2} C_4$$

equiv. série

$$R_4 = \frac{R_2}{R_1}\left(\frac{R_3}{1+\omega^2 R_3^2 C_3^2}\right)$$

$$C_4 = \frac{R_1}{R_2}\left(C_3 + \frac{1}{\omega^2 R_3^2 C_3}\right)$$

Ponte de WIEN (continuação)

em equilíbrio:

$$R1\,(R4 + Z_{C4}) = R2\,\frac{Z_{C3}\,R3}{R3 + Z_{C3}}$$

$$R1\,R4 - j\,\frac{R1}{\omega C4} = R2\,\frac{\frac{1}{j\omega C3}\,R3}{R3 + \frac{1}{j\omega C3}}$$

$$R1\,R4 - j\,\frac{R1}{\omega C4} = \frac{R2\,R3}{1 + j\omega R3 C3}$$

$$R1\,R4 - j\,\frac{R1}{\omega C4} = \frac{R2\,R3\,(1 - j\,\omega R3\,C3)}{1 + \omega^2 R3^2\,C3^2}$$

$$\boxed{R1\,R4 - j\,\frac{R1}{\omega C4} = \frac{R2\,R3}{1 + \omega^2 R3^2 C3^2} - j\,\frac{\omega\,R2\,R3^2\,C3}{1 + \omega^2 R3^2\,C3^2}}$$

equiv. série:

$$R4 = \frac{R2\,R3}{R1\,(1 + \omega^2 R3^2 C3^2)}$$

$$C4 = \frac{R1\,(1 + \omega^2 R3^2 C3^2)}{\omega^2\,R2\,R3^2\,C3} = \frac{R1}{R2}\left(\frac{1}{\omega^2 R3^2 C3} + C3\right)$$