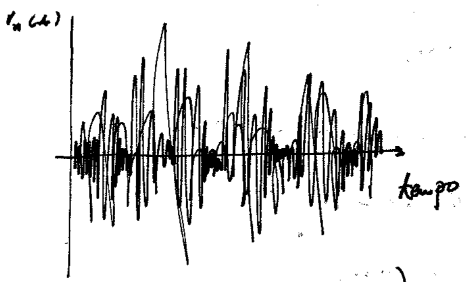
TESTE 3 JUNHO 10 H

# AULA 6 (INTERFACIAMENTO) — RUÍDO INTRÍNSECO




$$\bar{v}_n(t) = \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

valor médio é zero

(Potência DC equivalente)

valor RMS

$$v_n(rms) = \left[\frac{1}{T}\int_0^T v_n^2(t)\,dt\right]^{\frac{1}{2}}$$

$$i_n(rms) = \left[\frac{1}{T}\int_0^T i_n^2(t)\,dt\right]^{\frac{1}{2}}$$

Potência de ruído normalizada $R = 1\,\Omega$

$$P = \frac{v_n^2(rms)}{1\,\Omega} = 1\,\Omega \times i_n^2(rms)$$

exemplo: 1 sinal com 1 mV (rms) dissipa a mesma potência que um sinal dc de 1 mV.

Fontes de ruído independentes

Supor 2 fontes de ruído $v_1(t)$, $v_2(t)$

$$v_o(t) = v_1(t) + v_2(t)$$

$$v_o^2(rms) = \frac{1}{T}\int_0^T \left(v_1(t) + v_2(t)\right)^2 dt$$

$$= v_1^2(rms) + v_2^2(rms) + \frac{2}{T}\int_0^T v_1(t)\cdot v_2(t)\,dt$$

definição de coeficiente de correlação

$$C = \frac{\frac{1}{T}\int_0^T v_1(t)\cdot v_2(t)\,dt}{v_1(rms)\cdot v_2(rms)}$$

$$v_o^2(\;) = v_1^2(rms) + v_2^2(rms) + 2C\, v_1(rms)\cdot v_2(rms)$$

Se $v_1(t)$ e $v_2(t)$ estatisticamente independentes

$$\boxed{v_o^2(rms) = v_1^2(rms) + v_2^2(rms)}$$

Exemplo

$v_1(rms) = 10\mu V$

$v_2(rms) = 5\mu V$

$v_o(rms) = ?$

$$v_o^2(rms) = (10^2 + 5^2)(\mu V)^2 = 125\,(\mu V)^2$$

$$v_o(rms) = 11.2\,\mu V$$

(notar apenas uma subida de 10%.)

Quanto deve baixar $v_1$ para $v_o(rms)$ se manter em $10\mu V$?

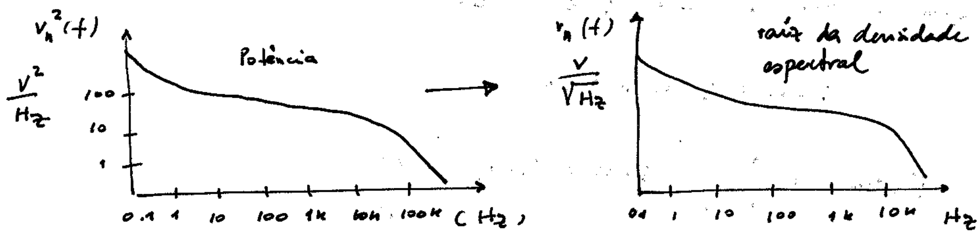
$$(10\mu V)^2 = v_1^2(rms) + (5\mu V)^2$$

$$v_1(rms) = 8.7\,\mu V$$

(notar que basta reduzir $v_1$ de 13% ⇒ Para reduzir o ruído concentrar o esforço no sinal mais ruidoso!)

## Densidade espectral do ruído

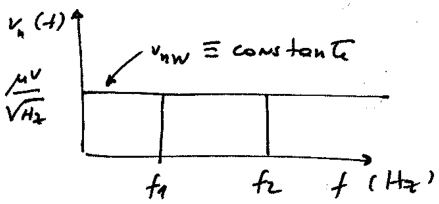
(Johns pág 186)



A potência de ruído numa determinada gama de frequências $f_1$ a $f_2$ é dada por

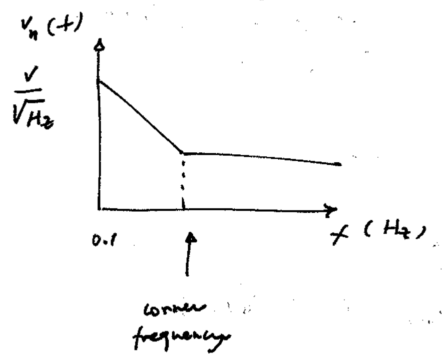
$$v_n^2(rms) = \int_{f_1}^{f_2} v_n^2(f)\,df$$

## Ruído branco




$$v_n^2(rms) = v_{nW}^2 (f_2 - f_1)$$

## Ruído 1/f (ou flicker)




(frequência de torção)

$$v_n^2(f) = \frac{K_V^2}{f}$$

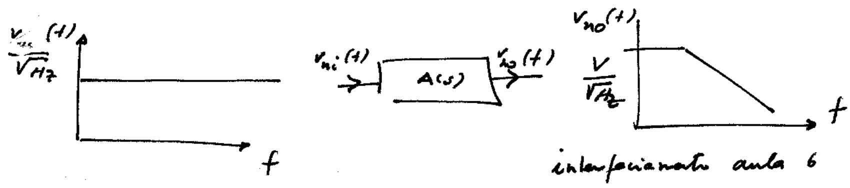
$$v_n(f) = \frac{K_V}{\sqrt{f}}$$

$K_V$ - constante de proporcionalidade

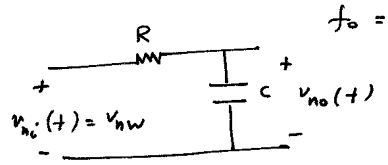
## Ruído filtrado


$$v_{no}^2(f) = |A(j2\pi f)|^2\, v_{ni}^2(f)$$

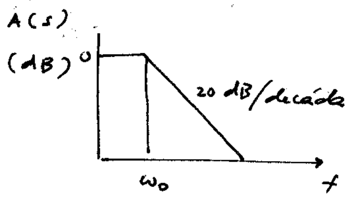
$$V_{no}(f) = |A(j2\pi f)|\, v_{ni}(f)$$



## largura de banda equivalente de um filtro de 1ª ordem (RC)




$$f_0 = \frac{1}{2\pi RC}$$

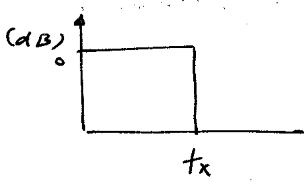



$$A(s) = \frac{1}{1 + \frac{s}{\omega_0}}$$

$$A(j2\pi f) = \frac{1}{1 + j\frac{f}{f_0}}$$

$$v_{no}^2(rms) = \int_0^{\infty} \frac{v_{nw}^2}{1 + \left(\frac{f}{f_0}\right)^2} df = v_{nw}^2 \, f_0 \left[ \operatorname{arctg} \frac{f}{f_0} \right]_0^{\infty} = \frac{v_{nw}^2 \, \pi f_0}{2}$$

filtro abrupto equivalente (brickwall filter)




$$v_{no}^2(rms) = \int_0^{f_x} v_{nw}^2 \, df = v_{nw}^2 f_x$$

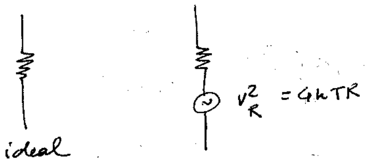
Comparando

$$\boxed{f_x = \frac{\pi}{2} f_0}$$ largura de banda equivalente

( sendo $f_0 = \frac{1}{2\pi RC}$ vem $f_x = \frac{1}{4RC}$ )

## modelos de ruído para componentes electrónicos (pag 199 Johns)

### resistências




k – constante de Boltzman ($= 1.38 \times 10^{-23}\, JK^{-1}$)

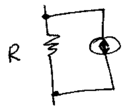
T – temperatura absoluta

– ruído branco

– origem: agitação térmica dos electrões

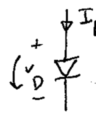
$R = 1\,k\Omega \Rightarrow v_R = 4.06\, nV/\sqrt{Hz}$

equivalente de Norton:




$$I_R^2 = \frac{v_R^2}{R^2} = \frac{4kT}{R}$$

### diodos

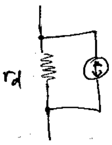



$$I_D = I_S\, e^{\frac{qv_D}{kT}}$$

$$\frac{1}{r_d} = \frac{\partial I_D}{\partial r_D} = \frac{q}{kT} I_D$$

$$r_d = \frac{kT}{q I_D}$$

modelo pequeno sinal



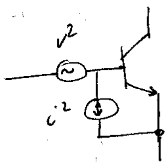

$I_D^2 = 2q I_D \iff$


$v_N^2 = 2kT\, r_d$

– ruído branco

– origem: natureza discreta dos portadores de carga (electrões)

### transistor bipolar (zona activa)




resistência na base

$$v^2 = 4kT\left(r_b + \frac{1}{2g_m}\right)$$

R transcondutância

$$g_m = \frac{\partial I_c}{\partial r_{be}} = \frac{I_c}{V_T}$$

constante de proporcionalidade

$$i^2 = 2q\left(I_B + \frac{k_v I_B}{f}\right)$$

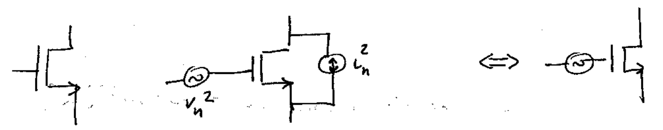
ruído 1/f

flicker noise

origem:

irregularidades nas junções

## MOSFET




região activa

$$v_n^2 = \frac{K_V}{W L C_{ox} f}$$

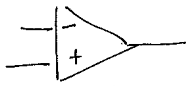
$$i_n^2 = 4kT\left(\frac{2}{3}\right) g_m$$

↙ ruído branco

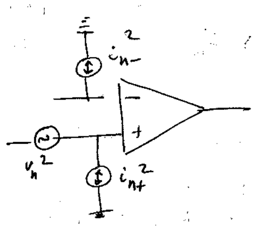
$$v_n^2 = \frac{4kT\left(\frac{2}{3}\right)}{g_m} + \frac{K_V}{W L C_{ox} f}$$

↙ ruído branco ↙ $\frac{1}{f}$ noise

## OPAMP



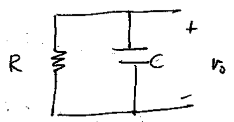

ideal

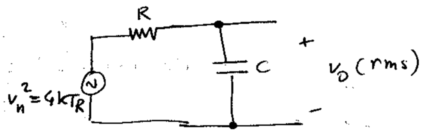



condensadores e inductores – não geram ruído (mas armazenam ruído)

## Densidade espectral de potência de um circuito RC

modelo de ruído



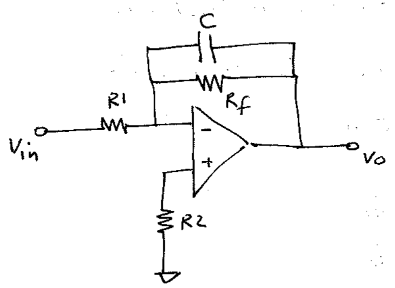



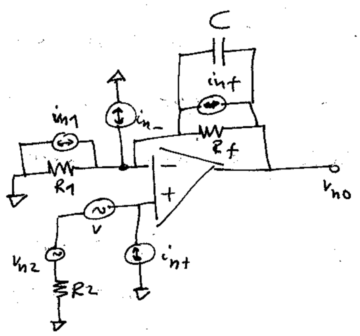

$$v_o^2(rms) = v_n^2 \frac{\pi}{2} f_o = v_n^2 \frac{\pi}{2} \frac{1}{2\pi RC}$$

$$v_o^2(rms) = 4kTR \frac{\pi}{2} \frac{1}{2\pi RC} = \frac{kT}{C}$$

– potência de ruído nos terminais de um condensador independente de R e proporcional a $\frac{1}{C}$

exemplo: OPAMP em montagem inversora



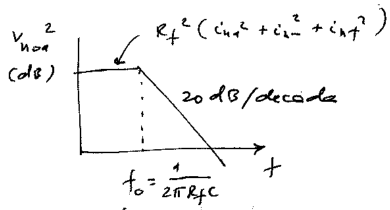

$$v_{no1}^2 = \left[ i_{n1}^2 + i_{n-}^2 + i_{n+}^2 \right] \left| R_f \,||\, C \right|^2$$

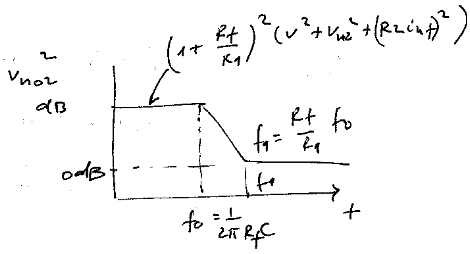
$$R_f \,||\, C = \frac{R_f \times \frac{1}{j\omega C}}{R_f + \frac{1}{j\omega C}} = \frac{R_f}{1 + j\omega R_f C}$$

$$v_{no1}^2 = \left[ i_{n1}^2 + i_{n-}^2 + i_{n+}^2 \right] \frac{R_f^2}{1 + 4\pi^2 f^2 (R_f C)^2}$$

$$v_{no2}^2 = \left[ v^2 + v_{n2}^2 + (R2\, i_{n+})^2 \right] \left| 1 + \frac{R_f \,||\, C}{R_1} \right|^2$$

$$= \left[ v^2 + v_{n2}^2 + (R2\, i_{n+})^2 \right] \left| 1 + \frac{R_f / R_1}{1 + j\omega R_f C} \right|^2$$




$$f_1 \Rightarrow \frac{\cancel{R_f} / R_1}{2\pi f_1 \cancel{R_f} C} = 1$$

$$\Leftrightarrow \quad f_1 = \frac{1}{2\pi R_1 C}$$

$$f_1 = \frac{R_f}{R_1} f_0$$

exemplo: Calcule a potência de ruído total quando

$C_f = 160\ pF$, $R_f = 100\ k\Omega$, $R_1 = 10\ k\Omega$, $R_2 = 9.1\ k\Omega$

Assumir para o OP.AMP $v_n = 20\ nV/\sqrt{Hz}$, $i_n = 0.6\ pA/\sqrt{Hz}$

e a frequência para ganho unitário $f_T = 5\ MHz$

$$i_{nR}^2 = \frac{4kT}{R} \qquad i_{nf} = 0.406\ pA/\sqrt{Hz}$$

$$v_{nR}^2 = 4kTR \qquad i_{n1} = 1.28\ pA/\sqrt{Hz}$$

$$v_{n2} = 12.2\ nV/\sqrt{Hz}$$

$$v_{no1}^2(0) = \left(i_{n1}^2 + i_{nf}^2 + i_{n-}^2\right) R_f^2$$

$f = 0\ Hz$

$$= \left(0.406^2 + 1.28^2 + 0.6^2\right)\left(1 \times 10^{-12}\right)^2 (100k)^2$$

$$= \left(147\ nV/\sqrt{Hz}\right)^2$$

$$v_{no1}^2(rms) = v_{no1}^2(0)\,\frac{\pi}{2} f_0 = v_{no1}^2(0)\,\frac{\pi}{2}\,\frac{1}{2\pi R_f C}$$

$$= \left(147\ nV/\sqrt{Hz}\right)^2 \frac{1}{4}\,\frac{1}{(100k)(160pF)}$$

$$= (18.4\ \mu V)^2$$

$$v_{no2}^2(0) = \left(v_n^2 + v_{n2}^2 + (i_{n+} R_2)^2\right)\left(1 + \frac{R_f}{R_1}\right)^2$$

$$= \left(24.1\ nV/\sqrt{Hz}\right)^2 (11)$$

$$= \left(265\ nV/\sqrt{Hz}\right)^2$$

$$v_{no2}^2(rms) = \left(265\ nV/\sqrt{Hz}\right)^2 \frac{\pi}{2}\,\frac{1}{2\pi R_f C} + \left(24.1\ nV/\sqrt{Hz}\right)^2 \frac{\pi}{2}\left(f_T - f_1\right)$$

com $f_T = 5\ MHz$ e $f_1 = \frac{R_f}{R_1} f_0 = 100\ kHz$

$$v_{no2}^2(rms) = (74.6\ \mu V)^2$$

$$V_{n\,total}^2(rms) = (18.4\ \mu V)^2 + (74.6\ \mu V)^2 = (77\ \mu V)^2$$

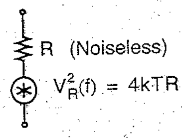
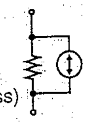
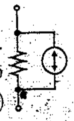
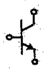
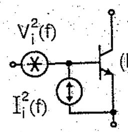
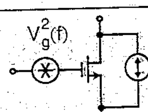
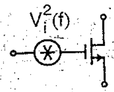
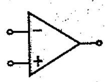
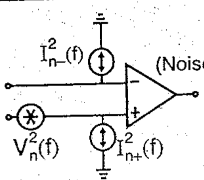
| Element | Noise Models | |
|---|---|---|
| Resistor   |   $V_R^2(f) = 4kTR$ |   $I_R^2(f) = \frac{4kT}{R}$ |
| Diode  (Forward biased) |  $r_d = \frac{kT}{qI_D}$ (Noiseless) $V_d^2(f) = 2kTr_d$ |  $r_d = \frac{kT}{qI_D}$ (Noiseless) $I_d^2(f) = 2qI_D$ |
| BJT  (Active region) |   $V_i^2(f) = 4kT\left(r_b + \frac{1}{2g_m}\right)$ $I_i^2(f) = 2q\left(I_B + \frac{KI_B}{f} + \frac{I_C}{\lvert\beta(f)\rvert^2}\right)$ | |
| MOSFET  (Active region) |   $V_g^2(f) = \frac{K}{WLC_{ox}f}$ $I_d^2(f) = 4kT\left(\frac{2}{3}\right)g_m$ |   $V_i^2(f) = 4kT\left(\frac{2}{3}\right)\frac{1}{g_m} + \frac{K}{WLC_{ox}f}$ Simplified model for low and moderate frequencies |
| Opamp  |   $V_n(f)$, $I_{n-}(f)$, $I_{n+}(f)$ — Values depend on opamp — Typically, all uncorrelated | |

**Fig. 4.11** Circuit elements and their noise models. Note that capacitors and inductors do not generate noise.