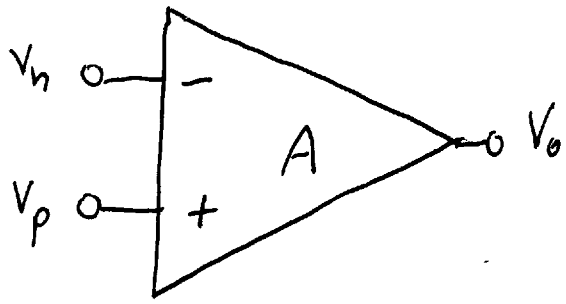
# CHAPTER 4

# OPAMP CIRCUITS

The operational amplifier is a very versatile component that can be used in many ways, ranging

- high gain amplifiers
- active filters
- signal generators
- comparators
- Schmitt triggers
- tension followers
- Analog computer: integrators / differentiators / exp/log
- current controlled voltage sources.




$$V_0 = A\,(V_p - V_n)$$

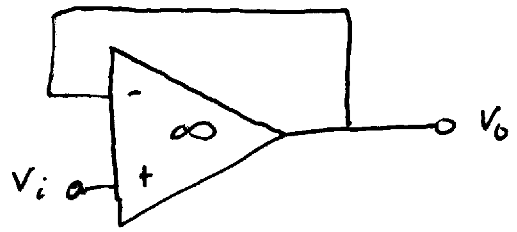
An ideal opamp has

1) infinite gain, $A = \infty$
2) infinite input resistance $r_i = \infty$
3) zero output resistance $r_o = 0$

1a) Because $A = \infty$, $V_p - V_n = V_o / A = 0$

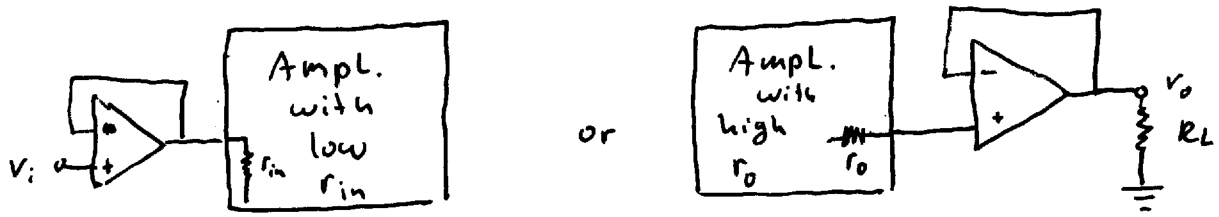
$\Rightarrow \quad V_p = V_n$ !

The easiest circuit we can make with an op-amp is a $\boxed{\text{tension follower}}$:

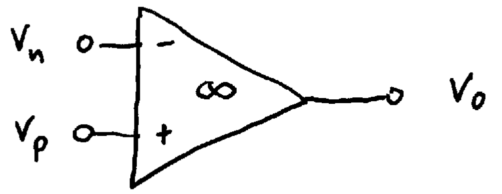



with 100% feedback ($\beta = 1$) and assuming an infinite open-loop gain ($A = \infty$) it is easy to show that $v_o = v_i$.

The advantage of this circuit is that it has very high input resistance and low output resistance. This is usefull when we cannot load a high-ohmic source or when we need to drive a low-ohmic input stage.



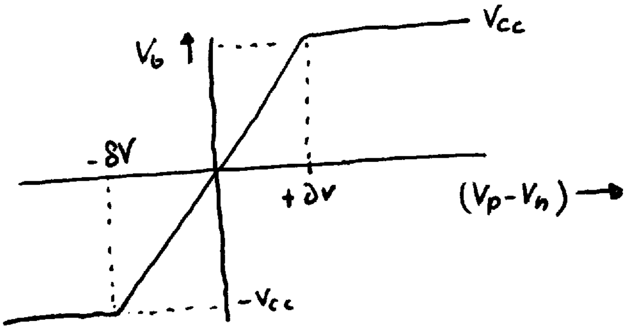

A $\boxed{\text{comparator}}$ is a circuit that has either $V_{cc}$ or $-V_{cc}$ (the supply voltage) as output, depending on the difference in the input terminals.




$V_o = +V_{cc}$ if $V_p > V_n$

$V_o = -V_{cc}$ if $V_p < V_n$

For op-amps with non-infinite gain $A$, there is a region of input voltages that don't give $\pm V_{cc}$ at the output




It is easy to show that $\partial V = V_{cc}/A$ and is of the order of $10\,V/10^5 \sim 100\,\mu V$ for practical opamps.

A widely used comparator op amp is the "311" which has a high switching speed $-V_{cc} \leftrightarrow +V_{cc}$

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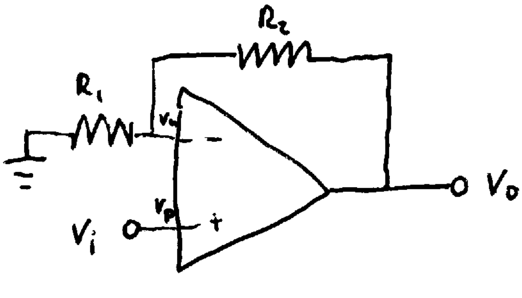
A $\boxed{\text{non-inverting amplifier}}$ can be made by introducing (negative) feedback to the amplifier.



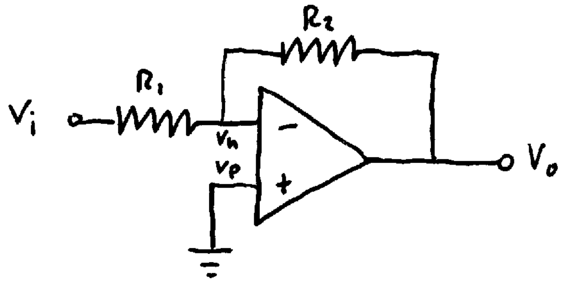

To calculate the gain:

$V_n = V_p = V_i$ (rule 1a of p. 1)

$$V_n = \frac{R_1}{R_1 + R_2} V_o$$

$$\Longrightarrow \quad \frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1}$$

An $\boxed{\text{inverting amplifier}}$ is equal to the one above, but with the signal connected to $R_1$:



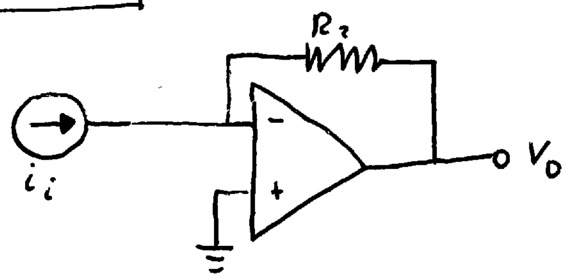

$V_n = V_p = 0\ V$ and that is why the negative terminal in this scheme is ofter called <u>virtual</u> <u>ground</u>.

A current of $I_i = \frac{V_i}{R_1}$ is drawn from the source and (since $r_{in} = \infty$), this current <u>must</u> go through $R_2$.

The output voltage thus becomes

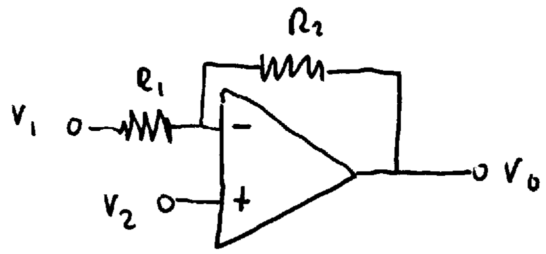
$$V_o = \frac{R_2}{R_1} \cdot V_i \implies \frac{V_o}{V_i} = \frac{R_2}{R_1}$$

The same idea is also used in a $\boxed{\text{current-to-voltage converter}}$




$$\frac{V_o}{i_i} = R_2$$

For a $\boxed{\text{differential amplifier}}$ we connect both inputs:




using superposition we can easily see that

$$V_0 = \frac{R_2}{R_1} V_2 - \frac{R_1 + R_2}{R_1} V_1 \approx \frac{R_2}{R_1}(V_2 - V_1) \qquad (\text{for } R_1 << R_2)$$

we have to always bear in mind that a high gain for $R_2 >> R_1$ ($\beta$ is very small) will limit the bandwidth. see p.7 of chapter 3:

$$f_c = f_0 (1 + A_0 \beta)$$

with $f_0$ the open-loop bandwidth and $A_0$ the open-loop gain. In fact, the gain-bandwidth product.

$$GBW = \frac{V_o}{V_i} \times f_c$$

is constant. $\left( \frac{V_o}{V_i} = \frac{A}{1+A\beta},\ f_c \approx f_0(1 + A\beta) \right),\ GBW = A f_0$

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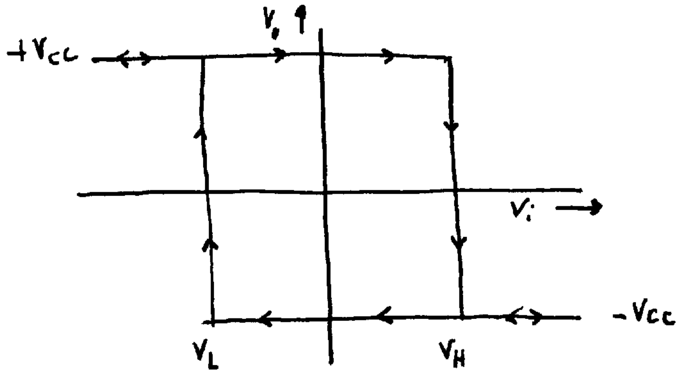
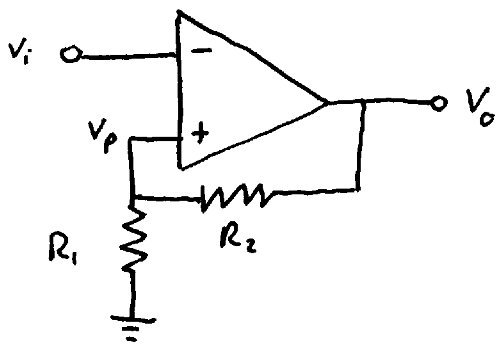
A $\boxed{\text{SCHMITT TRIGGER}}$ is like a comparator, but it has a memory. The output of the schmitt trigger not only depends on the input voltage, but also on the history.




To achieve, positive feedback is used




$$V_p = \frac{R_1}{R_1+R_2} V_o$$

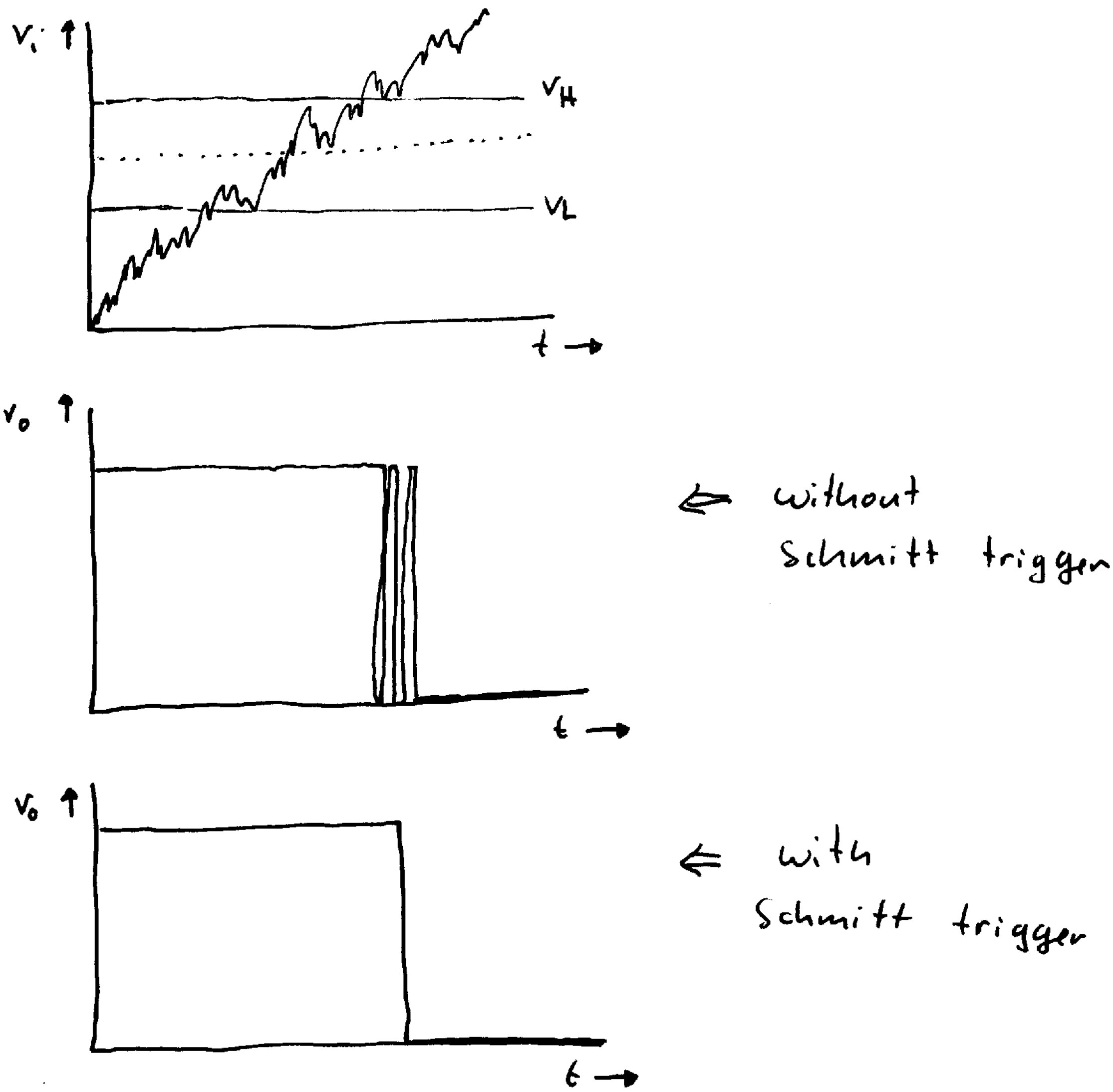
If $V_o = +V_{cc}$, then $V_p = +\frac{R_1}{R_1+R_2} V_o$. Now, if we increase $V_i$ from $-\infty$ upwards, there comes a point where $V_i > V_p$ and thus, since the circuit works as a comparator, $V_o$ becomes $-V_{cc}$. This makes $V_p = -\frac{R_1}{R_1+R_2} V_o$. To make the output commute again, we have to lower $V_i$ to this value.

Thus

$$V_L = -\frac{R_1}{R_1+R_2} V_{cc}$$

$$V_H = +\frac{R_1}{R_1+R_2} V_{cc}$$

The advantage of a Schmitt trigger lies in the fact of eliminating noise.




Imagine switching on the lights in a room based on a detector. ...

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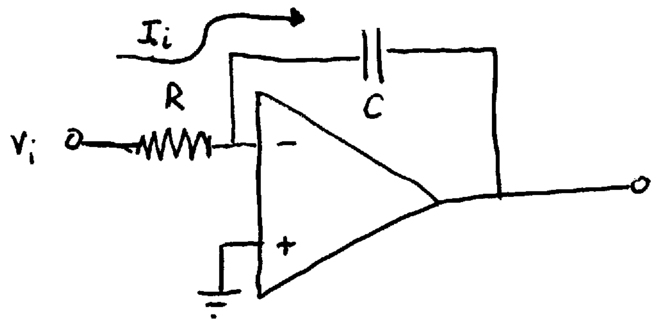
$\boxed{\text{Integrators}}$ and $\boxed{\text{Differentiators}}$ are made by putting a condensator in the feedback loop.




⇐ Integrator

The resistor translates the voltage to a current $I_i = V_i/R$. This current cannot enter the opamp and thus is used to charge the capacitor C.

The amount of charge in C (assuming at $t=0$ discharged, $Q(t=0)=0$) is equal to the integrated current:

$$Q(t) = \int_0^t I_i(t)\,dt = \int_0^t \frac{V_i(t)}{R}\,dt$$

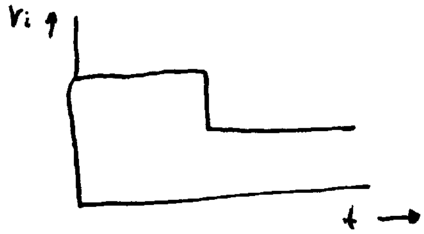
The voltage drop induced by this charge is

$$\Delta V_c = Q/C$$

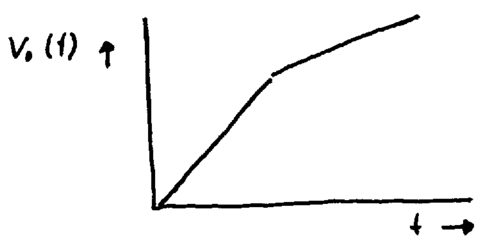
Since one side of the capacitor is connected to virtual ground, the voltage drop is equal to $-V_o$.

Thus

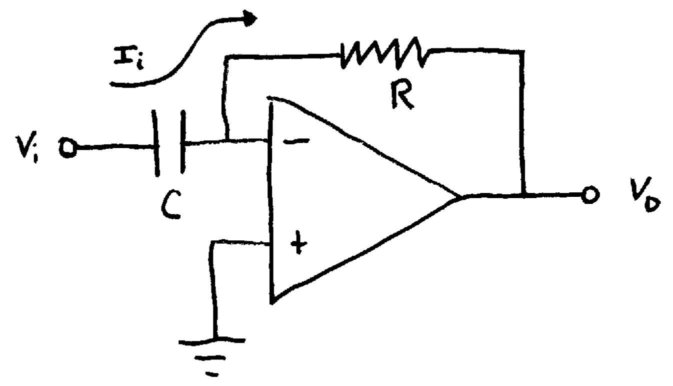
$$V_o(t) = -\frac{Q(t)}{C} = -\frac{1}{CR}\cdot\int_0^t V_i(t)\,dt$$




$\Longrightarrow$



To make a $\boxed{\text{differentiator}}$, we exchange the resistance and capacitor




The input current $I_i$ is equal to

$$I_i = C \cdot \frac{dv_i}{dt}$$

This current is translated to voltage by R (note that one side of the resistor is at virtual ground and the input resistance of the op-amp is infinite; all current goes through R).

$$V_o(t) = -CR \frac{dv_i(t)}{dt}$$

In both cases, the differentiator and integrator the signals at the output might be limited by the power supply. $V_o$ cannot be larger that $\pm V_{cc}$. In the case of the integrator it means that, for instance, DC signals at $v_i$ can only be integrated up to a certain time. For the differentiator it means that signals cannot change too fast.

with the integrator and differentiator circuits we can build $\boxed{\text{analog computers}}$. For instance for solving differencial equations.

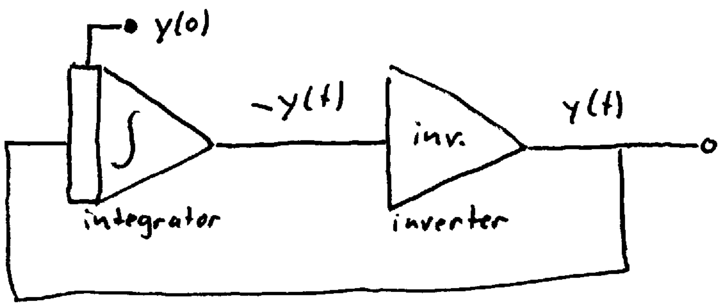
Take for example the differential equation

$$\dot{y} = y \quad \text{with} \quad y(0) = 1 \qquad \left( \dot{y} = \frac{dy}{dt} \right)$$

This is equivalent to (integrating on both sides)

$$y(t) = y(0) + \int_0^t y(\tau)\, d\tau$$

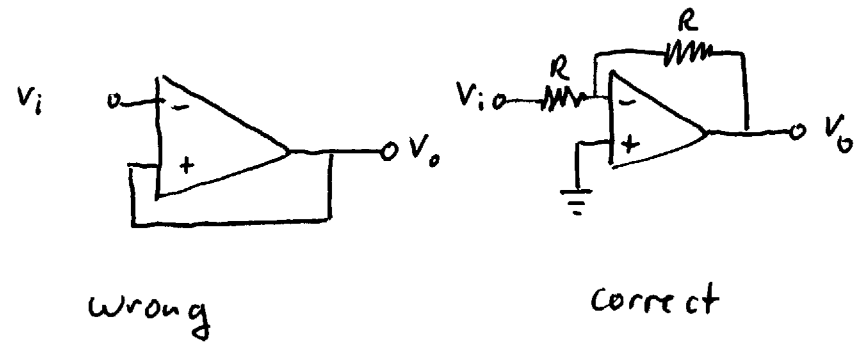
with our op-amp circuits this becomes




Note 0: The integrator also inverts the signal!

Note 1: the ⌐• y(0) part is to load the inverter with the starting value (implies charging the capacitance instantaneously).

Note 2: The inverter can also easily be made of opamps. Why is it not the left circuit?

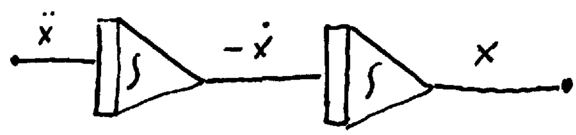


As another example:

$$\ddot{x} + 0.5\dot{x} + x = 4.0 \qquad x(0) = 0, \quad \dot{x}(x) = 1$$

$$\dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d^2x}{dt^2}$$

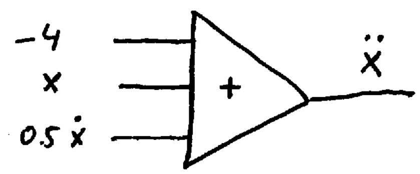
1) Starting with assuming $\ddot{x}$ exists:




2) Rearrange the equation

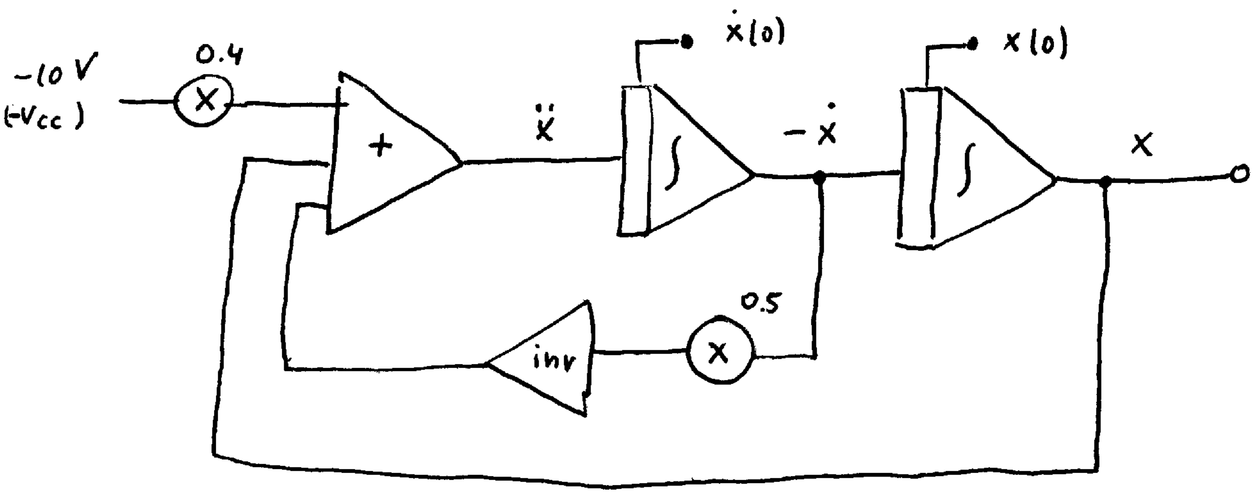
$$\ddot{x} = -0.5\dot{x} - x + 4.0$$

This is equal to



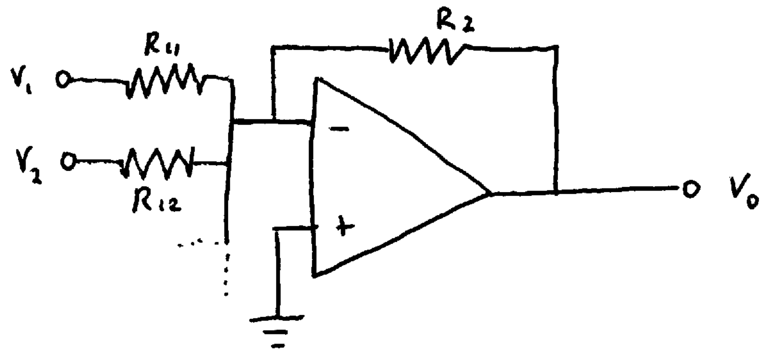

(The summing opamp also inverts!)

3) Now connecting the two parts ("procedures")



* The multipliers $\otimes$ are easily made with opamps. when it is a constant multiplication factor.

* The adder can also be made with opamps.

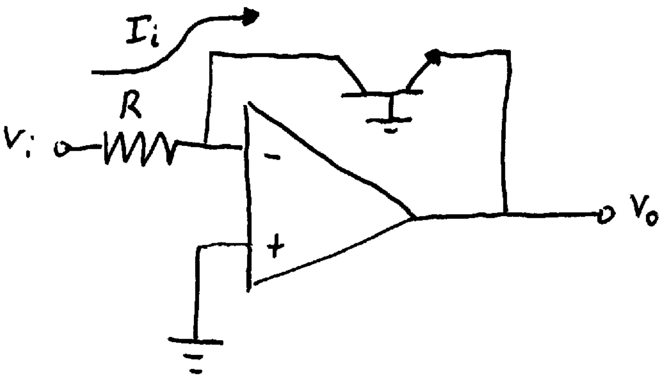



$$V_0 = -\left(\frac{R_2}{R_{11}}V_1 + \frac{R_2}{R_{12}}V_2 + \dots\right)$$

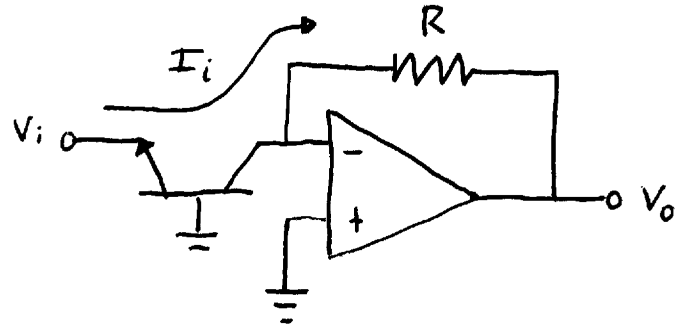
* The circuit on the previous page can be simplified because there are too many inverters, but this is a small detail.

The circuit on the previous page is an example of how a damped oscillation can be calculated using analog electronics. An example of a damped oscillation is a guitar string or a bell. Thus, the above circuit generater musical signals. The first electronic synthesizers were made in this way. We can imagine that the values at $\ulcorner^{\bullet}\ \dot{x}(0)$ and $\ulcorner^{\bullet}\ x(0)$ were put there by a touch of a key, where the note (frequency) is also determined by the integration constants (RC) of the integrators.

Other "mathematical" circuits are the exponential and logaritmic amplifiers




logarithmic amplifier




exponential amplifier

In the logarithmic amplifier: the input current $I_i = V_i/R$ passes through the transistor. Since $V_C = 0$ (virtual ground) and $V_B = 0$, we can use the diode equation of Ebers - Moll.

$$I_c = V_i/R = I_o \left\{ \exp\left( \frac{V_{BE}}{V_T} \right) - 1 \right\}$$

Ignoring the $-1$ and using $V_o = -V_{BE}$ we see that

$$V_o = -V_T \ln \left( \frac{V_i}{I_o R} \right)$$

Note that $V_o$ has to be positive.

For the exponential amplifier it is not difficult to show that

$$I_i \approx - I_0 \exp\left(-\frac{V_i}{V_T}\right)$$

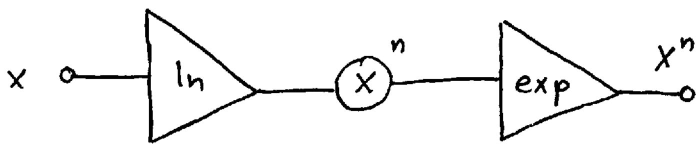
$$V_0 = -I_i R = I_0 R \exp\left(-\frac{V_i}{V_T}\right)$$

(note: $V_i$ has to be negative)

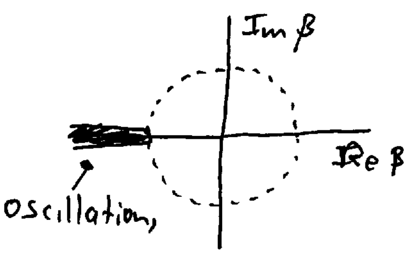
Combining logarithmic and exponential (anti-logarithmic) amplifiers we can make calculations of the type

$$y = x^n$$

$$y = \exp(n * \ln(x))$$



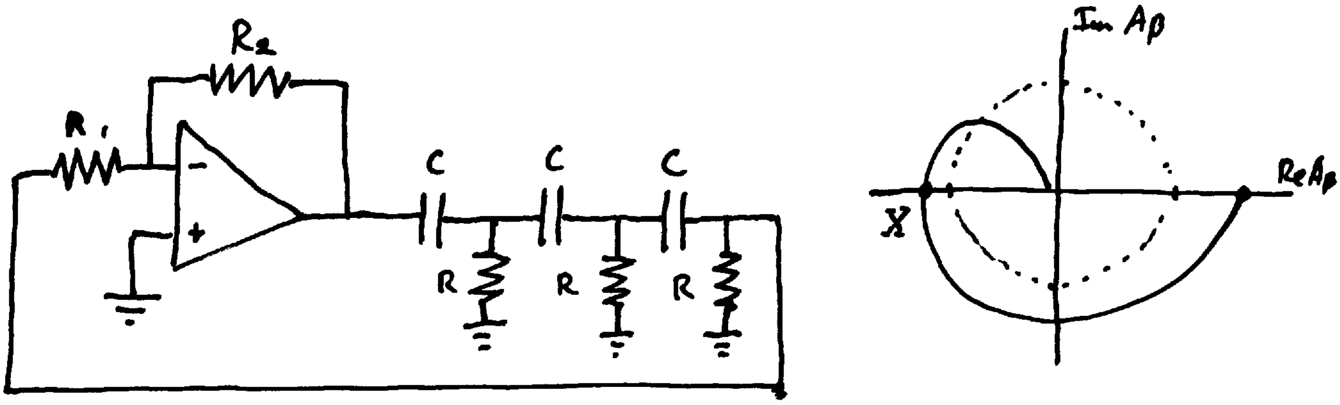

$\boxed{\text{Oscillators}}$. In chapter 3 we saw how to avoid oscillations. Now we will use the same analysis to make oscillations happen. We need




$A\beta$ to become real and $<-1$ for some frequency (see Nyquist ← plot here).

Assuming a flat A, without poles and phase changes, we can see that 3 poles are needed in $\beta$ to achieve that.

For example




$$\beta = \frac{R^3}{(R^3 - 5R/\omega C) + j\,((1/\omega C)^3 - 6R^2/\omega C)}$$

In the oscillation frequency, at X, the imaginary part of $\beta$ is zero.

$$\left(\frac{1}{\omega c}\right)^2 = 6R^2 \quad \Rightarrow \quad \omega = \frac{1}{\sqrt{6}}\,RC$$

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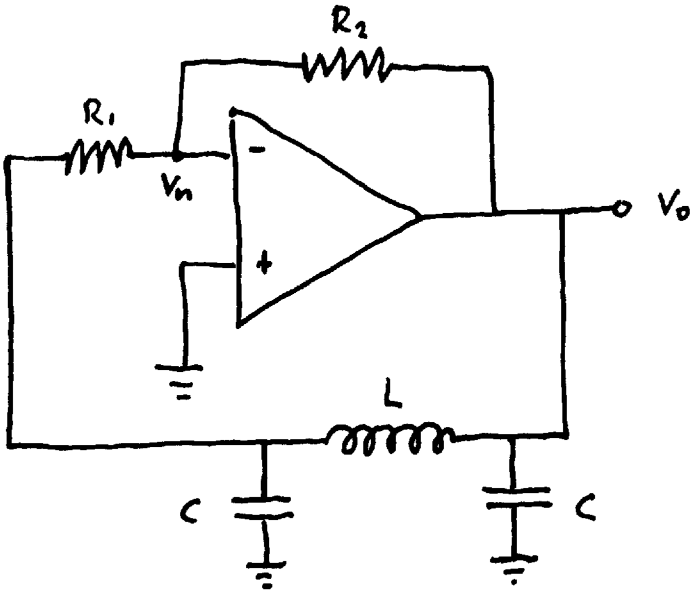
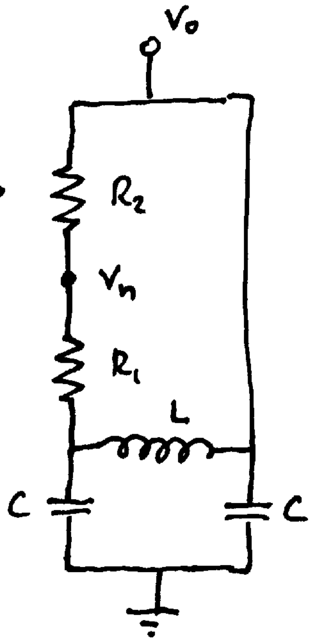
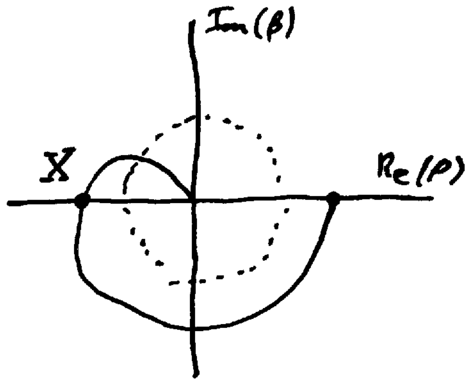
Another example is the Colpitts Oscillator. It consists of an opamp with negative feedback, so it needs also three poles. In this case they are made by two capacitances and one coil.




Colpitts Oscillator

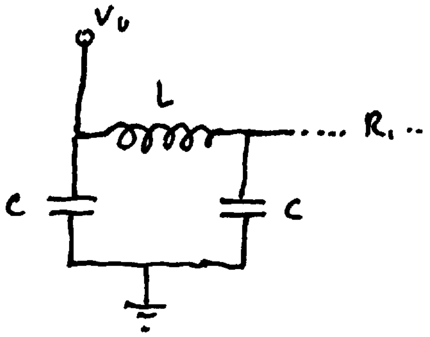
The feedback $\beta$ loop can be visualized as →




It is not easy to calculate $\beta = \frac{V_n}{V_o}$, but we can make a simplification:

At X the feedback must be real (imaginary part = 0). Plus, if we assume $R_1$ to be large, it must mean that the impedance from the output of the amplifier looking into the CLC bridge must be real:




$$Z = 1/j\omega C \parallel (j\omega L + 1/j\omega C)$$

$$= \frac{\frac{1}{j\omega C}\left(j\omega L + \frac{1}{j\omega C}\right)}{\frac{1}{j\omega C} + j\omega L + \frac{1}{j\omega C}}$$

$$Z = \frac{(L/c - 1/\omega^2 C^2)}{j\left(-\frac{1}{\omega c} + \omega L - \frac{1}{\omega c}\right)} \quad \text{must be real}$$

$$\Rightarrow \quad \frac{-2}{\omega c} + \omega L = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2}{LC}}, \quad f = \frac{\omega}{2\pi} = \sqrt{\frac{1}{2\pi^2 LC}}$$

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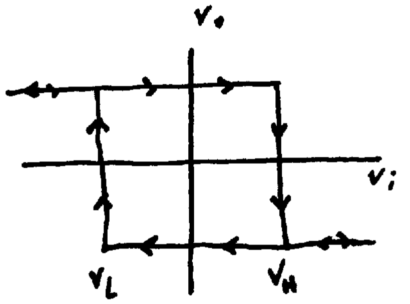
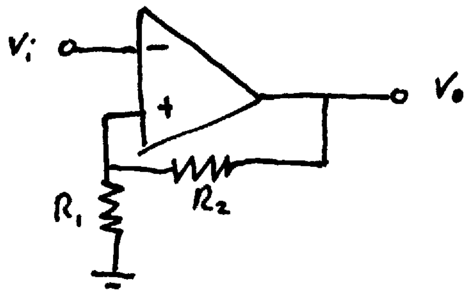
A $\boxed{\text{memory element}}$ can be made of a Schmitt trigger




For $V_i = 0$, there are two possible states at the output $+V_{cc}$ and $-V_{cc}$, depending on the history. Thus, for $V_i = 0$, we can read the memory element. To program the element, a voltage higher than $V_H = \frac{R_1}{R_1 + R_2} V_{cc}$ should be set at $V_i$. This makes the output to $-V_{cc}$ (logical "0"). To program a logical "1", a voltage below $V_L = -\frac{R_1}{R_1+R_2} V_{cc}$ should be set at the input.

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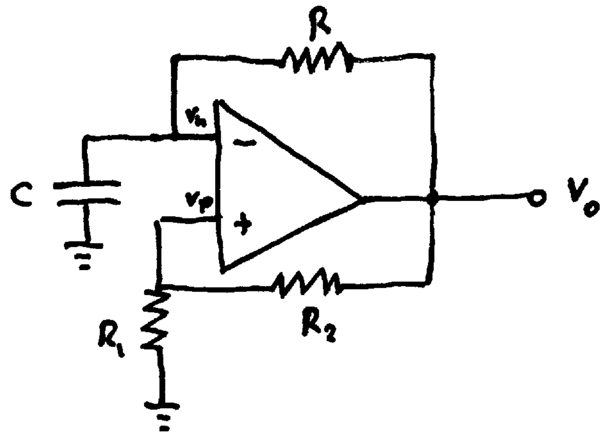
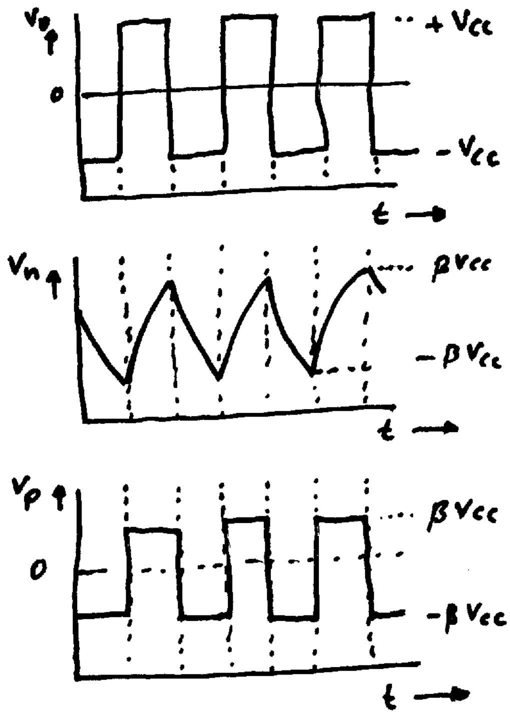
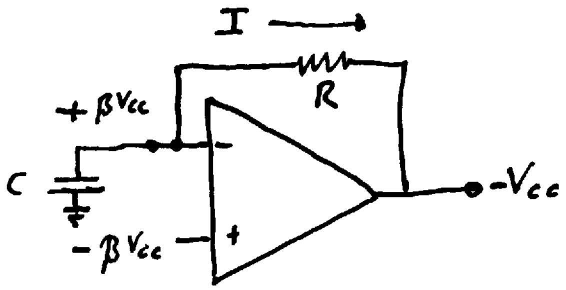
A memory element is a bistable element. It can take either of two output values and is stable. Contrasting this is a $\boxed{\text{Astable Multivibrator}}$ which is astable at both possible output values $\pm V_{cc}$



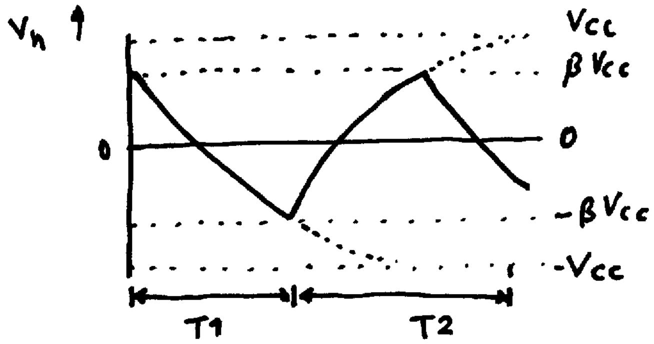

$$\beta = \frac{R_1}{R_1 + R_2}$$

Imagine $V_o$ at $-V_{cc}$. $V_p$ is then at $-\frac{R_1}{R_1+R_2} \cdot V_{cc}$ $= -\beta V_{cc}$. Imagine $V_n$ starts at $+\beta V_{cc}$.




Through the resistance R will initially $(t=0)$ go a current $I = (\beta V_{cc} - (-V_{cc}))/R$.

This current will uncharge C! Because of this, $V_n$ is lowering (Note that thus I is constantly reduced and is not constant) It is thus exponentially decaying from $+\beta V_{cc}$ to $-V_{cc}$, with an RC time of $R \times C$. However, when it reaches $-\beta V_{cc}$ it becomes lower than $V_p$. In this case the output commutes $-V_{cc} \rightarrow +V_{cc}$ and a reverse cycle starts.




How long does it take to charge the capacitor?

1) Starts at $+\beta V_{cc}$
2) Aiming at $-V_{cc}$

$$V_n = -V_{cc} + (\beta+1) V_{cc} \exp(-t/\tau)$$

3) The relaxation time is $\tau = RC$ $\rightarrow$ $V_n = -V_{cc} + (\beta+1) V_{cc} \exp(-t/RC)$

4) Stops at $V_n = -\beta V_{cc}$

$$\Longrightarrow \quad T_1 = RC \ln\left(\frac{1+\beta}{1-\beta}\right)$$

In a similar way we can show that $T2 = T1$ and in total

$$T = 2\,RC \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$f_{osc} = \frac{1}{2\pi T}$$