CHAPTER 3

FEEDBACK

ch. 8 of Sedra

Feedback is the concept of putting part of the output signal back at the entrance input of the amplifier or circuit.

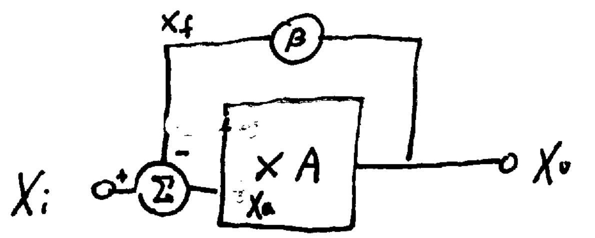
There are various advantages to this :

* stabilize the gain (independence on β, etc)
* change output and input resistances
* extend the bandwith

There are also negative effects such as

* instability (oscillations)

The basic feedback circuit consists of an amplifier, with a gain A and a feedback loop, which put part, β, of the output back at the input. For negative feedback this is subtracted at the input :



(We consider here only <u>negative</u> feedback .
It is not so difficult to see that positive feedback results in a runaway signal).

For negative feedback like in the figure above, it can be shown that

(1) $X_o = A X_a$ (X can be voltage or current !)

(2) $x_f = \beta x_o$

(3) $x_a = x_i - \cancel{R}x_f = x_i - \beta x_o$

(3) into (1) :

$$x_o = A (x_i - \beta x_o)$$

$$\boxed{\frac{x_o}{x_i} = \frac{A}{1 + A\beta}}$$ ⟸ NEGATIVE FEEDBACK

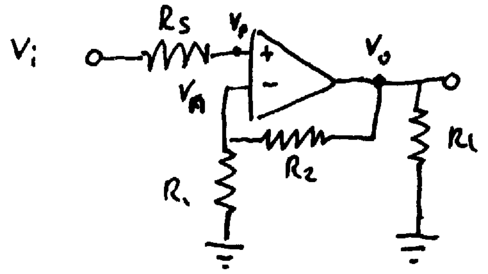
$A\beta$ is called the loopgain

When the amplifier A is an ideal amplifier, $A = \infty$ and the amplification becomes

$$\frac{x_o}{x_i} = \frac{1}{\beta}$$

This is an interesting result; the gain of an ideal amplifier with feedback is determined by the feedback loop.

Operational amplifiers have very large gain (of the order of 10^5) and the gain of op-amp's is therefore controlled by the feedback. As an example



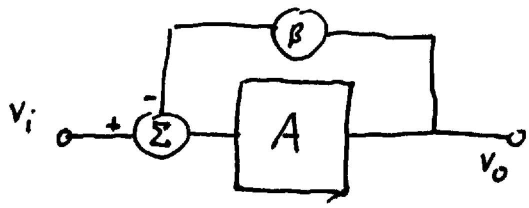
op-amp: $A = 10^4$
$r_{in} = \infty$
$r_{out} = 0$

$R_2 = 10\,k\Omega$
$R_1 = 1\,k\Omega$

- The feedback is from V_o to V_n and is supplied by a voltage divider R_1, R_2:

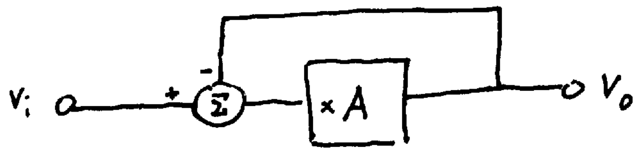
$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1\,k\Omega}{1\,k\Omega + 10\,k\Omega} =$$

- The open-loop gain is the gain from v_i to V_o, with the feedback disconnected. This is thus 10^4
- Because $r_{in} = \infty$, $i_i = 0$ and $V_p = V_i$



$$\left.\begin{array}{l} A = 10^4 \\ \beta = \end{array}\right\} \quad \frac{V_o}{V_i} = \frac{A}{1 + A\beta} =$$

In the other extreme case, when the amount of feedback is 100%



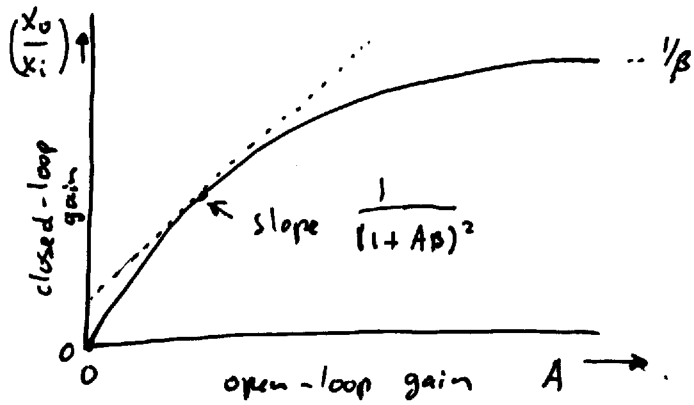
In this case the gain is

$$\frac{v_o}{v_i} = \frac{A}{1 + A\beta} = \frac{A}{1+A} \approx 1$$

The voltage at the output is exactly the voltage at the input. For this it is called a tension-follower the advantage is that it can convert a circuit with high output resistance into a circuit with low output resistance. (Imagine connecting 8 Ω speakers to our differential pair).

Desensitizing the gain. Not all amplifiers (op amps) coming from the factory are equal. They all have slightly different gain. Typically in the order of 5%. For some applications this is not accurate enough. The gain can be stabilized by feedback

$$\frac{d\left(\frac{x_o}{x_i}\right)}{dA} = \frac{1}{(1+A\beta)^2}$$



$$\frac{d\left(\frac{x_o}{x_i}\right)}{dA\left(\frac{x_o}{x_i}\right)} = \frac{1}{(1+A\beta)^2} \Big/ \frac{A}{(1+A\beta)}.$$

The <u>relative</u> change in gain is thus related to the relative change in gain of the open circuit:

$$\frac{d\left(\frac{x_o}{x_i}\right)}{\left(\frac{x_o}{x_i}\right)} = \frac{dA}{A} \times \frac{1}{(1+A\beta)}.$$

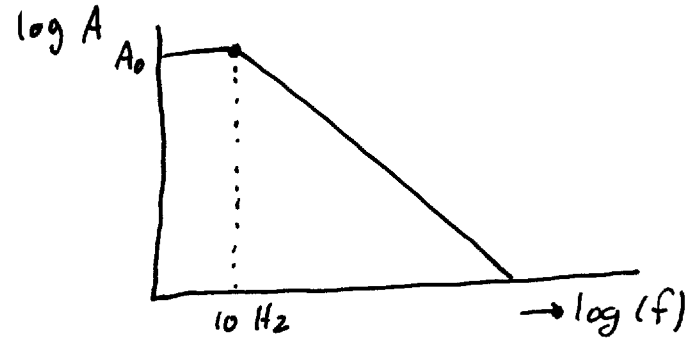
Thus, the effect of variations of different amplifiers is reduced by a factor $\frac{1}{1+A\beta}$. The gain is much more stable than the open circuit amplifier. an "error" in gain of the amplifier of $\frac{dA}{A} = 5\%$ has only a relative error of $\frac{1}{1+A\beta} \cdot 5\%$ in the final circuit.

The price that is payed is a reduced gain:

From A to $\frac{A}{1+A\beta}$

Bandwidth extension

An amplifier with feedback has an increased bandwidth. As an example, take a typical op-amp.



with a cut-off frequency at 10 Hz. The transfer function of this is

$$A(s) = \frac{A_0}{1 + s/\omega_0} \qquad \begin{aligned} \omega_0 &= 2\pi \times 10 \text{ Hz} \\ A_0 &= 10^5 \text{ (typical)} \end{aligned}$$

using this op-amp in an amplifier with feedback β:

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{A_0/(1+s/\omega_0)}{1 + A_0\beta/(1+s/\omega_0)}$$

$$= \frac{A_0/(A_0\beta+1)}{1 + s/(\omega_0\,(1+A_0\beta))}$$

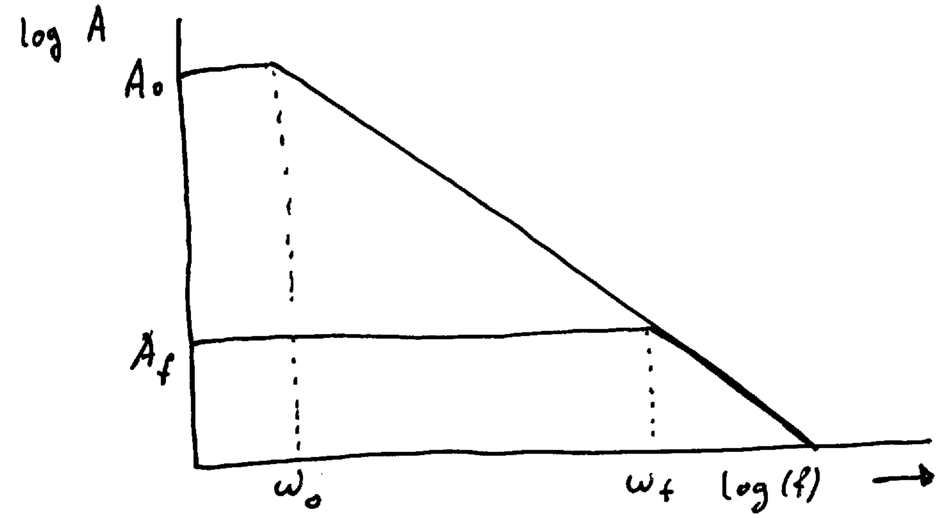
In other words:

The DC gain $= A_0/(A_0\beta+1)$

and the new cut-off frequency is

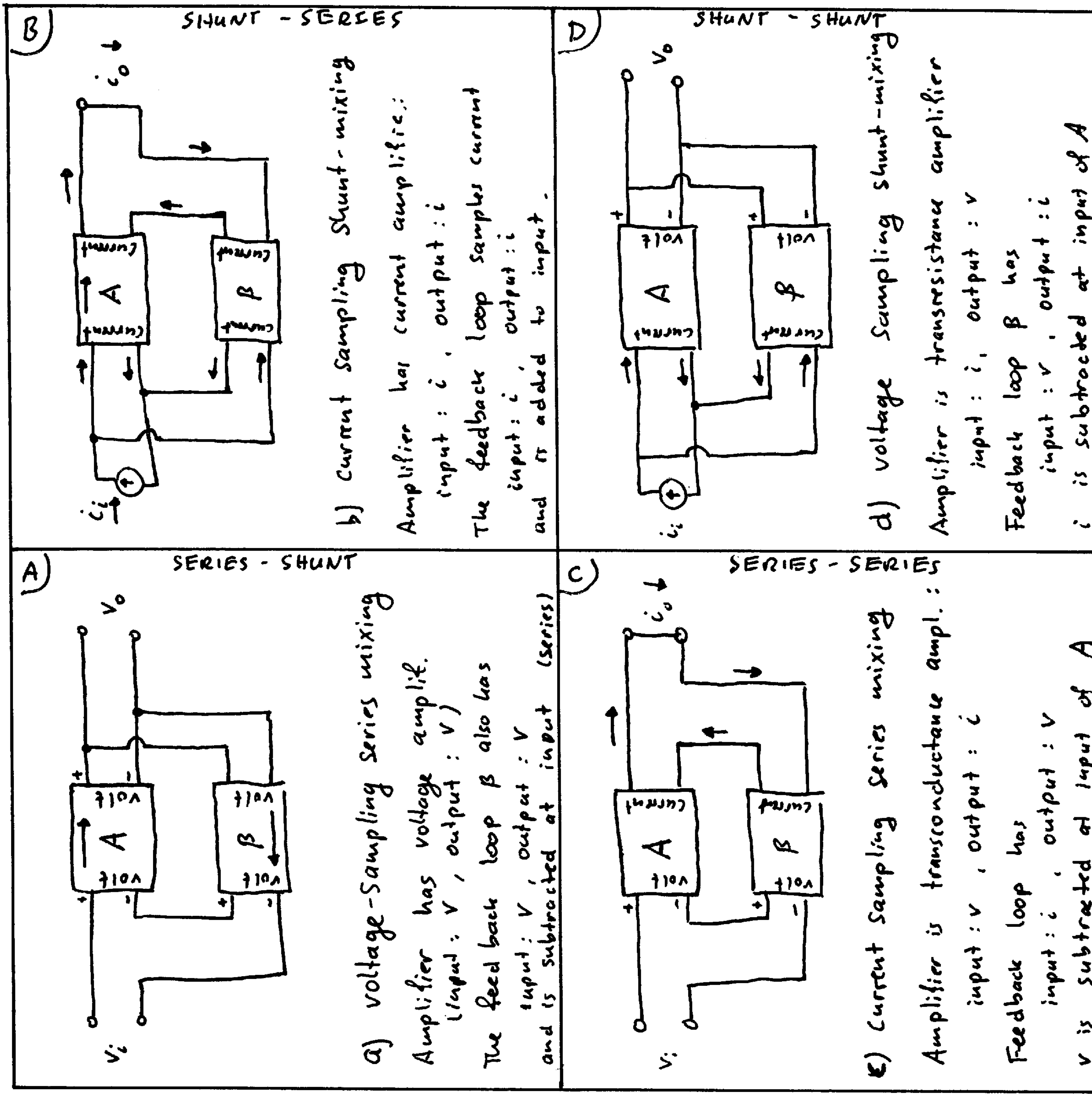
$$\omega_f = \omega_o (1 + A_o \beta)$$

By using feedback, the bandwidth of the amplifier is increased. The price that is payed is a reduced midband gain $A_o \rightarrow A_f = \frac{A_o}{1 + A_o \beta}$



Topologies

In the examples we used above, the output of the amplifier was a voltage signal and this was partially fed-back with a factor β and added to the input as a voltage signal. This topology is called voltage-sampling series-mixing (A)

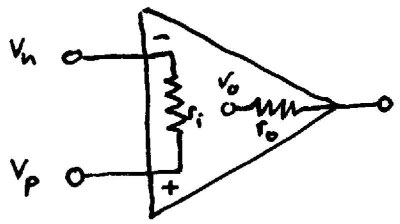


The other extreme is where the amplifier is a current amplifier. This current is sampled and with a factor β subtracted at the input (B)

The other two topologies are mixtures of current and voltage. (C) has a transconductance amplifier $(v \rightarrow i)$

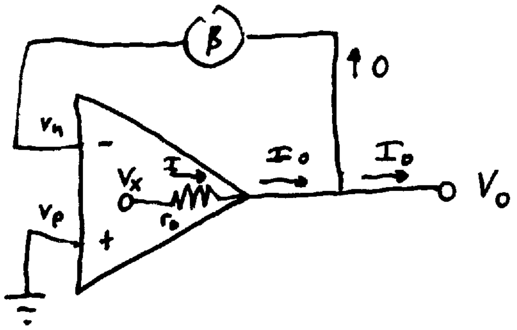
Output Resistance

The input and output resistances of the amplifier also change significantly. This is best illustrated on basis of an operational amplifier



r_i : input resistance
r_o : output resistance

To determine the effect on the output resistance we consider the input resistance infinite and connect (for the non inverting amplifier) V_p connected to ground.



(Because $r_i = \infty$ no current goes to V_n)

$$V_x = -A V_n$$

$$V_0 = V_x - I_0 r_0$$

$$V_n = \beta V_0$$

substituting gives

$$I_0 = V_0 \frac{1 + A\beta}{r_0}$$

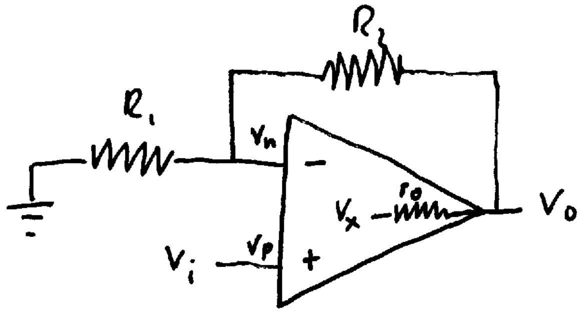
with the definition of output resistance

$r_o \equiv 1 \Big/ \frac{\partial I_o}{\partial V_o}$ we get

$$\boxed{r_{of} = \frac{r_o}{1 + A\beta}}$$

In other words : the output resistance is reduced by a factor $1 + A\beta$ by using feedback.
Since the amplifier is a linear circuit, for other voltages at the input (V_p) we will find the same result.
In the above example a feedback that is not loading the output was assumed. Normally the feedback loop is made with resistances and other conductive elements. In this case, there is a current being drawn from the output of the amplifier. When this amplifier has zero output resistance, the effect is nil; the amplifier can easily supply the current. If the output resistance r_o of the amplifier is not zero there is an effect on the open loop gain A.

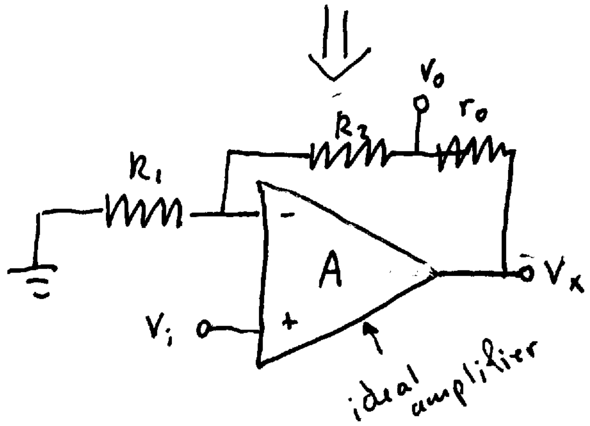
As an example : assume $r_o \neq 0$:



$$\beta_o = \frac{R_1}{R_1 + R_2}$$

$$\frac{v_o}{v_i} = \frac{A}{1 + A\beta_o} \leftarrow \text{ideally } (r_o = 0)$$

⇓



$$\frac{v_x}{v_i} = \frac{A}{1 + A\beta'}, \quad \beta' = \frac{R_1}{R_1 + R_2 + r_o}$$

$$\frac{v_o}{v_i} = \frac{v_x}{v_i} \cdot \frac{v_o}{v_x} = \frac{A}{1 + \frac{R_1}{R_1 + R_2 + r_o} \cdot A} \cdot \frac{R_1 + R_2}{R_1 + R_2 + r_o}$$

$$= \frac{A\alpha}{1 + A\alpha\beta_o}, \quad \text{with } \alpha = \frac{R_1 + R_2}{R_1 + R_2 + r_o}$$

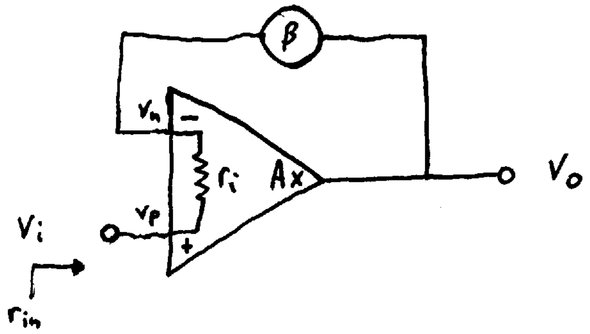
$$\beta_o = \frac{R_1}{R_1 + R_2}$$

In other words: because of the current loading of the feedback loop $(R_1 + R_2 < \infty)$ and the non-zero output resistance $(r_o \neq 0)$ the open-loop gain is reduced to

$$\boxed{A_1 = A \cdot \frac{R_1 + R_2}{R_1 + R_2 + r_o}}$$

Input Resistance

The input resistance can significantly increase when feedback is used



In this case it can be shown that :

$$V_o = (V_p - V_n)A = (V_i - V_n)A$$

$$V_n = \beta V_o = A\beta(V_i - V_n)$$

$$V_n = \frac{A\beta}{1+A\beta} V_i$$

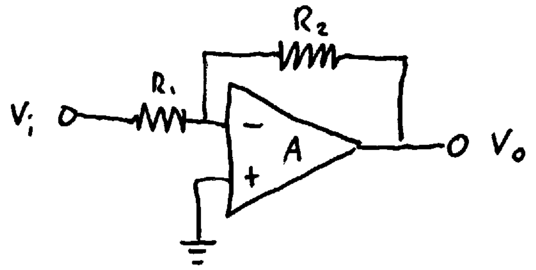
$$I_i = \frac{V_p - V_n}{r_i} = \frac{V_i - \frac{A\beta}{1+A\beta}V_i}{r_i} = \frac{1}{r_i} \cdot \frac{1}{1+A\beta} \cdot V_i$$

$$\boxed{r_{in} \equiv 1 \Big/ \frac{\partial I_i}{\partial V_i} = r_i\,(1+A\beta)}$$

Because of the feedback, the input resistance of this amplifier has increased a factor $1+A\beta$.

Not always is the increase in input resistance so dramatic

Example : Inverting amplifier with negative feedback.



a) what is the gain ($A_v = \frac{V_o}{V_i}$) of this circuit ?

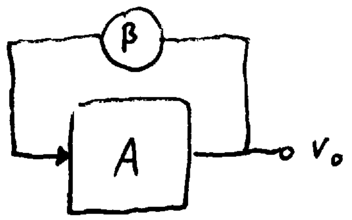
b) what is the output resistance ?

c) what is the input resistance ?

(Answ: $-\dfrac{A R_2}{A R_1 + (R_1+R_2)}$, $r_{out} = \dfrac{r_o}{1+A\beta}$, $r_{in} = R_1 + \dfrac{R_2}{1+A}$)

Feedback and Stability / oscillators

One of the negative aspects of feedback is that it can induce instabillities (oscillations) in the signals. This means that we can have an output signal at a certain frequency without having any input signal connected. It is clear that passive circuits (without amplification in any point) cannot oscillate, because the energy needed has to come from somewhere.



Generally speaking, a circuit with feedback β and open-loop gain A will oscillate when

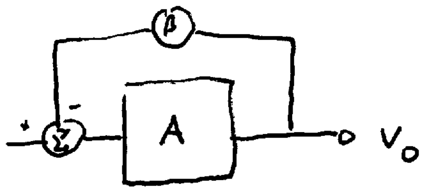
$A\beta = 1$. This is the $\boxed{\text{Barkhausen}}$ $\boxed{\text{Criterion}}$

For this case, any voltage at input of amplifier is added with factor 1 at amplifier and added again and again ... ab infinito! The output signal will be infinite. Note that often A and β depend on frequency, so it can occur that the output will be infinite for only certain frequencies

It will oscillate for all frequencies at which $A(f) \cdot \beta(f) = 1$

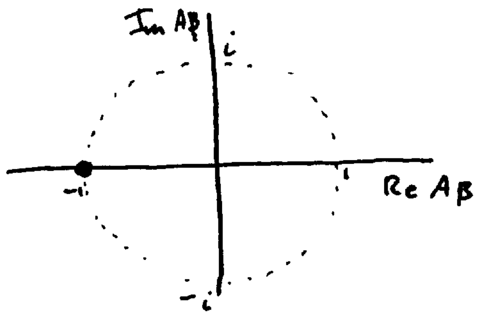
For our amplifier with <u>negative</u> feedback we found a gain of

$$\frac{X_o}{X_i} = \frac{A}{1 + A\beta}$$



Also here we can see that if $A\beta = -1$ then the output will be infinite for any tiny voltage at input. Normally, noise is present at any point of the circuit ($\sim \mu V .. mV$) and this will initiate the oscillations.

$A\beta = -1$ is equal to saying $|A\beta| = 1$ and the phase $\angle A\beta = 180°$, or in a phase diagram



⇐ The • gives the Barkhausen Criterion for amplifiers with negative feedback
The --- represents the points where $|A\beta| = 1$.

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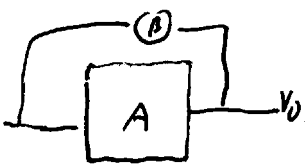
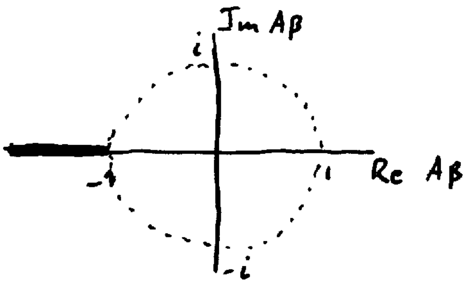
using the same reasoning (positive feedback): If the loop gain $A\beta > 1$, also oscillation will occur. For instance, for $A\beta = 2$ and starting with $1\ \mu V$ at the entrance, this voltage is fed back to the entrance with a factor 2 and added. Now we have $3\ \mu V$ at entrance. Then $3 + 6\ \mu V = 9\ \mu V$ ... etc.



In our picture of negative feedback:

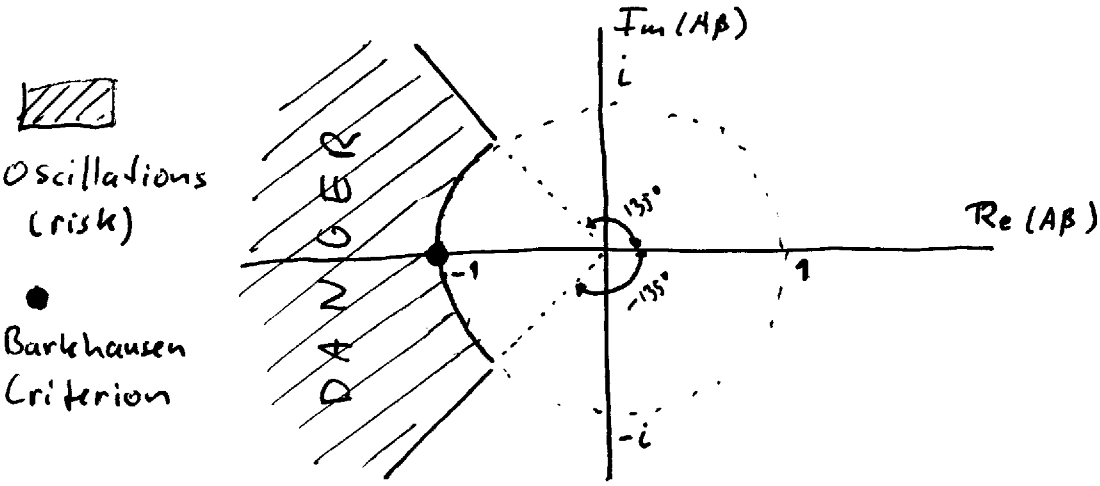


Oscillations when $|A\beta| > 1$

$\angle A\beta = 180°$

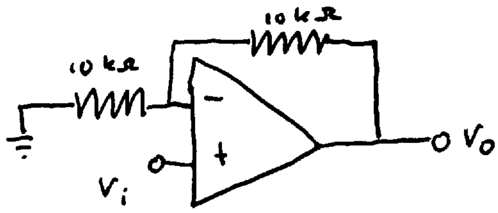
see ▬▬

Finally, a good electronic designing practice is to allow for a phase margin in the design. Normally 45° is used. With this, the final Nyquist plot (Im (Aβ) v.s. Re (Aβ)) becomes



Negative feedback. Nyquist plot.

As an example:



An op-amp with negative feedback. A has two poles, 10 Hz and 1 kHz. The DC gain is $+10^5$. Will this circuit oscillate? If so, at what frequencies? Assume $r_{in} = \infty$, $r_o = 0$, P.M. = 45°

$$\beta = \frac{R_1}{R_1+R_2} = 0.5 \qquad \phi = 0° \Rightarrow \phi(A\beta) = \phi(A)$$

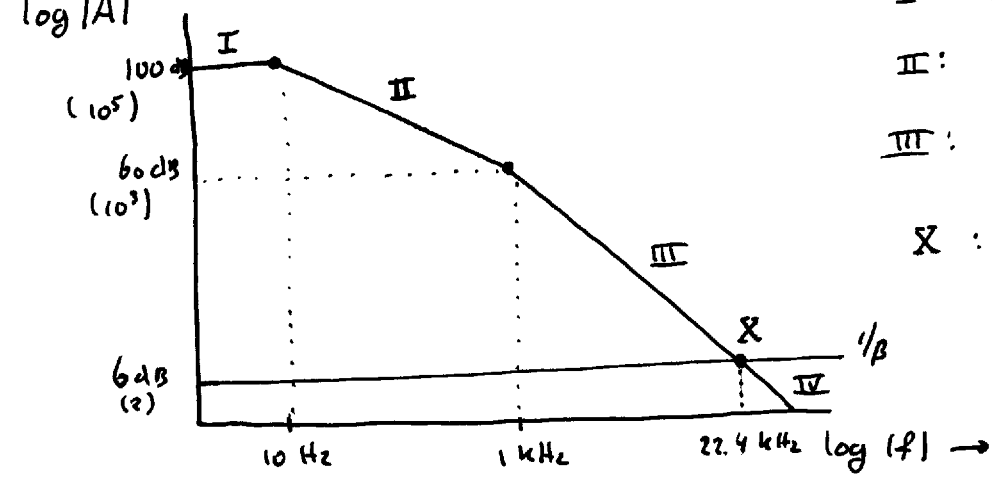
$$1/\beta = 2$$

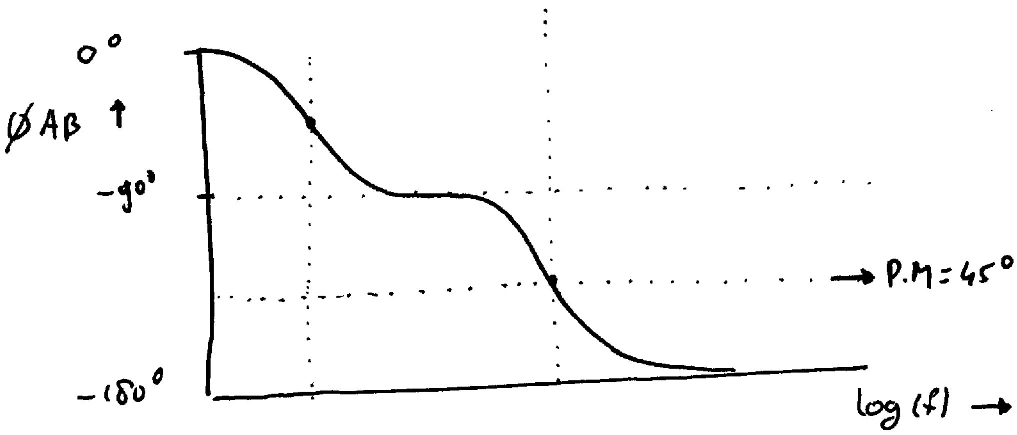
I: $A = 10^5$

II: $A = 10^5 \cdot \frac{10\text{ Hz}}{f}$

III: $A = 10^3 \cdot \frac{(1\text{ kHz})^2}{f^2}$

X : $|A| = 1/\beta \Rightarrow |A\beta| = 1$

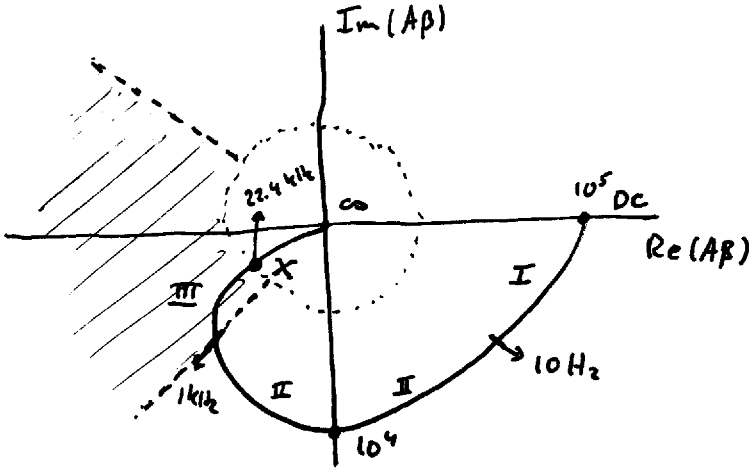




- When curve of $|A|$ crosses curve of $|1/\beta|$: $|A\beta| = 1$. we are somewhere on the circle in the Nyquist plot. But where?
- when $|A| < |1/\beta|$, the we are inside the circle and here the circuit is stable. zone IV
- In zones I and II, the phase is between 0° and -135° and the circuit is stable
- The problem is in zone III. Here the loop gain $|A\beta|$ is still larger than 1 and the phase is

inside the danger zone.

In a Nyquist plot:

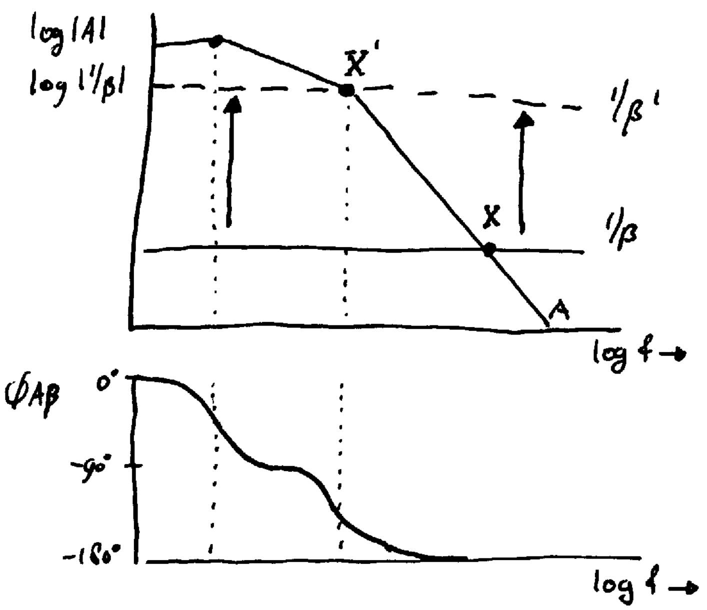


conclusion: The circuit runs the risk of oscillating in the frequency range 1 kHz — 22.4 kHz

The oscillations caused by feedback are not always bad. In fact, we can make oscillators in which the frequency is designed

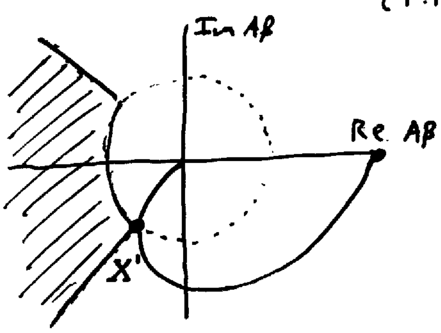
<u>How can the above circuit be stabilized?</u>

1) we could reduce the amount of feedback β



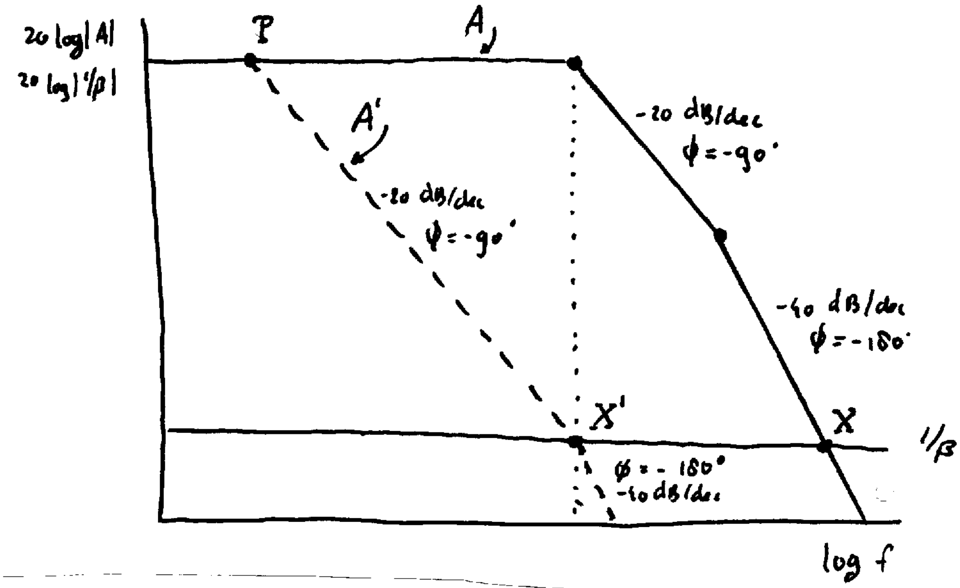
Marginally stable:

X' : $|A\beta| = 1$, $\phi A\beta = -135°$.
(P.M = 45°)



It is not difficult to show that a $\beta = 10^{-3}$ is needed to stabilize the circuit. For example with $R_1 = 100\,\Omega$ and $R_2 = 100\,k\Omega$.

2) <u>Frequency compensation</u> is another method for stabilizing the circuit, although it is rather done in the factory of the opamps. It consists of introducing a new pole in the open-loop gain A.



In the above figure: by introducing a new pole P at low frequencies the gain is reduced to below $1/\beta$ before the phase changes of the other poles take effect. To be sure this works for any β, a β of 1 has to be used in the calculation for the design of P

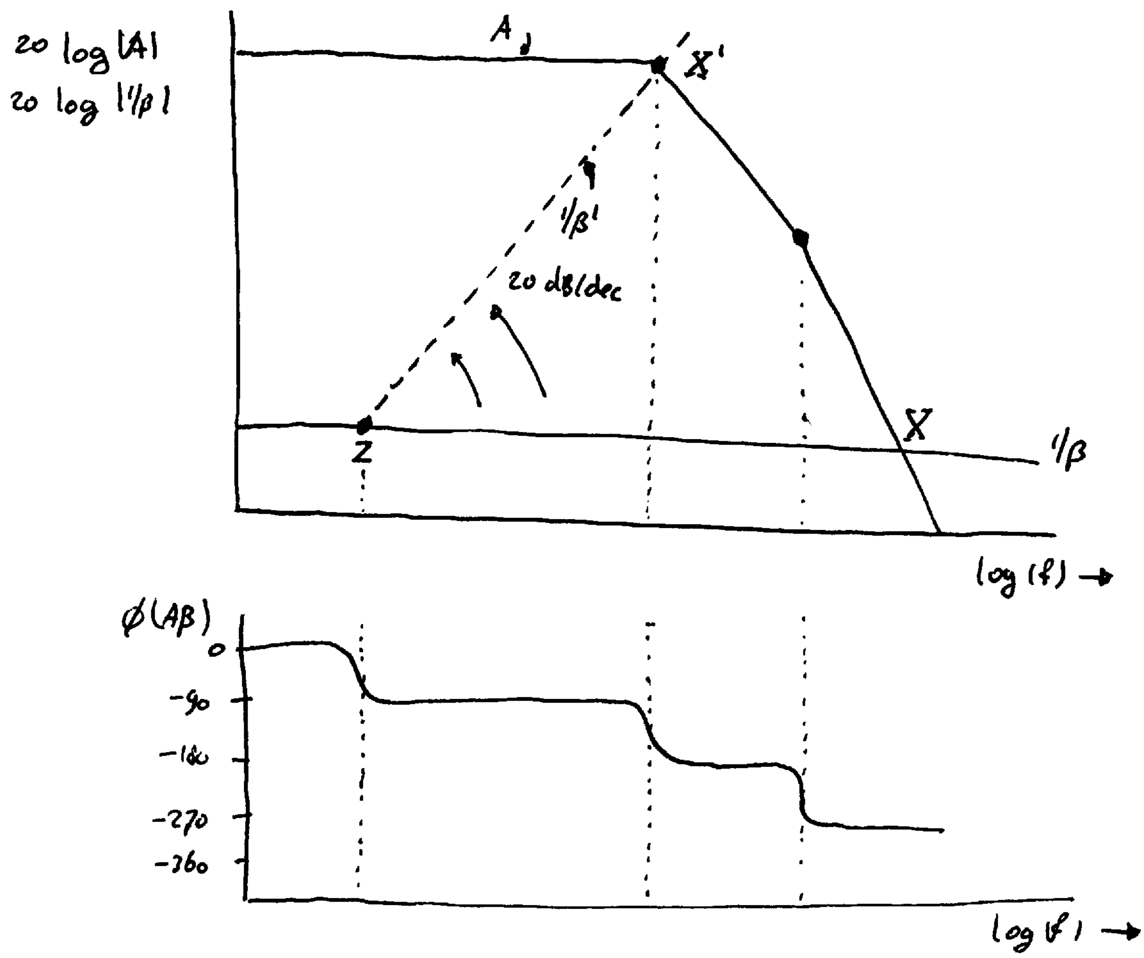
Most modern op-amps implement this technique and have extremely low cut-off frequencies (at around 10 Hz).

Question: what would be the frequency of the pole P needed in our op-amp in order to stabilize it for $\beta = 0.5$ and for $\beta = 1$

(answers: 2×10^{-4} Hz and 1×10^{-4} Hz)

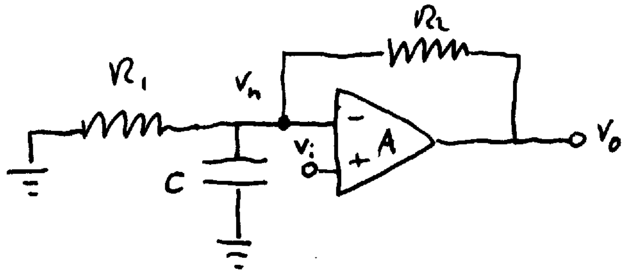
3 External compensation

In a similar way, introducing a pole in β (a zero in $1/\beta$) will result in stabilization of the circuit



By the zero z, the $1/\beta$ is pushed up to make it cross with A earlier, before the first pole of A.

The idea is to put an LPF in the feedback circuit β. For instance:



It is not difficult to see that

$$\beta \equiv \frac{V_n}{V_o} = \frac{R_1/(R_1+R_2)}{1 + (R_1 /\!/ R_2)\, s\, C} \quad \leftarrow \text{DC gain} = \frac{R_1}{R_1+R_2}$$

cut-off frequency

$$\frac{1}{2\pi\,(R_1 /\!/ R_2)\, C}$$

which we could have directly seen by considering the effective resistance seen by C, with V_o and V_i connected to ground.

Question: what will be the capacitance C needed to stabilize our circuit of page 16?

(Answer: $f_c = 2 \times 10^{-4}$ Hz, C = 160 mF)