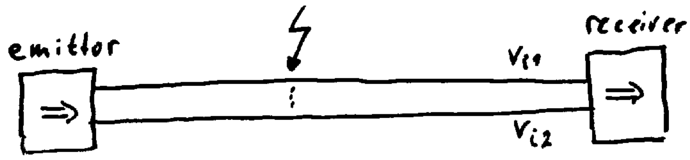
CHAPTER 1 : DIFFERENTIAL PAIR

A differential amplifier has two inputs and two outputs. The difference voltage at the output is proportional to the difference voltage at the input

$$V_{o2} - V_{o1} = (V_{i2} - V_{i1}) \cdot A$$

The advantage is obvious. For transmission lines, the noise is reduced



Whatever noise is introduced in the line, this noise is normally equal in both line. (The noise is "correlated"). Therefore, the difference is zero. $V_{i2} - V_{i1} = 0$. All noise will be rejected. Most communication lines work like this (twisted-pair network cables, etc.). The differential amplifier is therefore one of the most important electrical circuits.

An ideal diferencial amplifier thus reject all signals that appear on both terminals. We can define this in the common-mode rejection ratio (CMRR). This is the ratio of the gain at common mode

$$A_{cm} = \frac{V_o}{V_i} \qquad \text{with} \quad V_o = V_{o1} = V_{o2} \quad \text{and} \quad V_i = V_{i1} = V_{i2}$$

and the differencial gain

$$A_{dm} = \frac{V_{o2} - V_{o1}}{V_{i2} - V_{i1}}$$

Then

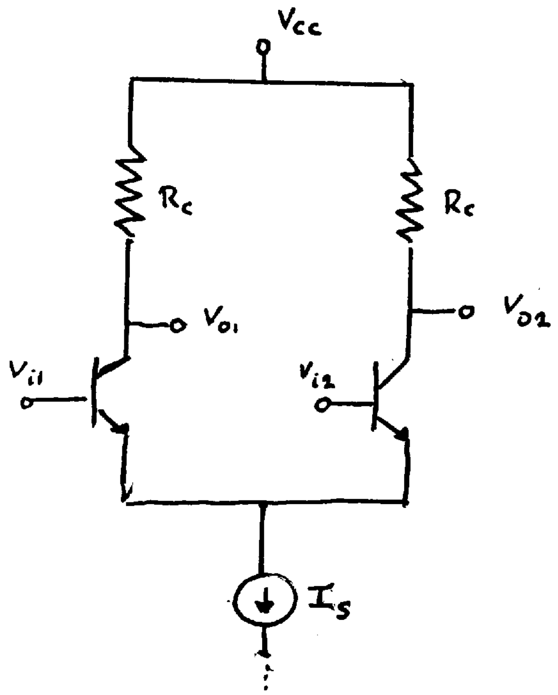
$$CMRR = \left| \frac{A_{dm}}{A_{cm}} \right|$$

This should be as high as possible. The CMRR is an important parameter to qualify a differencial amplifier. Other aspects are cost (number of components), power consumption and possibility to fabricate in integrated circuits.

One of the most popular is the differential pair (par diferencial).

Par Diferencial

(Sedra : ch 6
Bogart : ch 12
Moura : ch 1)



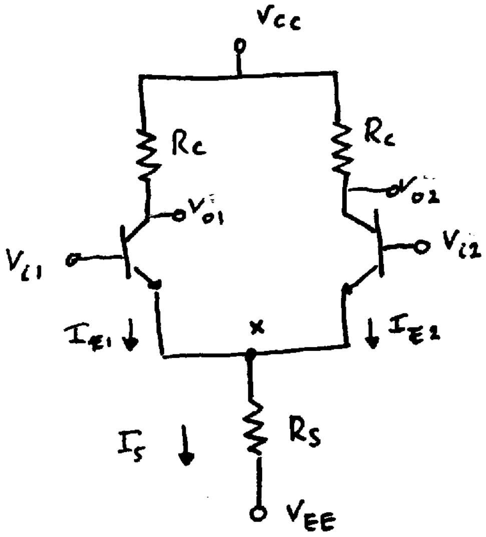
A basic differential pair consists of two (npn) transistors, a current source (I_s) and current-to-voltage converters R_c. We will calculate the gains (A_{cm} and A_{dm}) and CMRR for different versions of this basic model

1 : I_s defined by a simple resistance

2 : I_s a current source made with a transistor

3 : current-to-voltage converters made of pnp transistors

Model 1



⟸ simple DP. current source is a resistance. Why can we call it a current source? Because the DC voltage at x (the emitters of both transistors) is ~ −0.7 V (assuming small signals at V_{i1} and V_{i2} and no DC components)

$R_c = 3.3\,\mathrm{k\Omega}$, $R_s = 10\,\mathrm{k\Omega}$

$V_{cc} = +10\,V$, $V_{EE} = -10\,V$

Thus $I_s = (-0.7 - V_{EE}) / R_s$

<u>Polarization</u> : $V_{i1} = V_{i2} = 0\,V$

$V_x = -0.7\,V$. $I_s = (-0.7 + 10)/10\,\mathrm{k\Omega} = 0.93\,mA$.

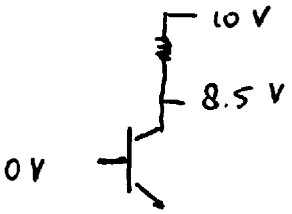
Because of symmetry : $I_{E1} = I_{E2} = \frac{1}{2} I_s = 0.465\,mA$.

$\beta = 100 \Rightarrow \alpha \approx 1$. $I_{c1} = I_{c2} = 0.465\,mA \Rightarrow V_{o1} = V_{o2} = 10\,V - 0.465\,mA \cdot 3.3k$

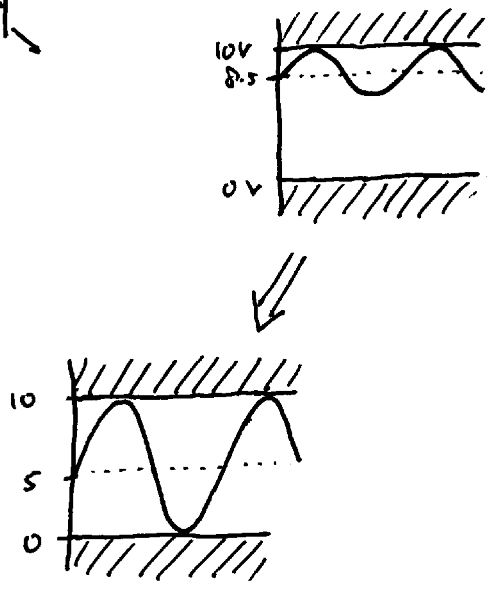
$= 8.47\,V$

$r_e = V_T / I_{E1} = 56\,\Omega$

Did we design this DP well? No!



The output can swing up to 10 V and down to ~ 0 V. This gives us a maximum



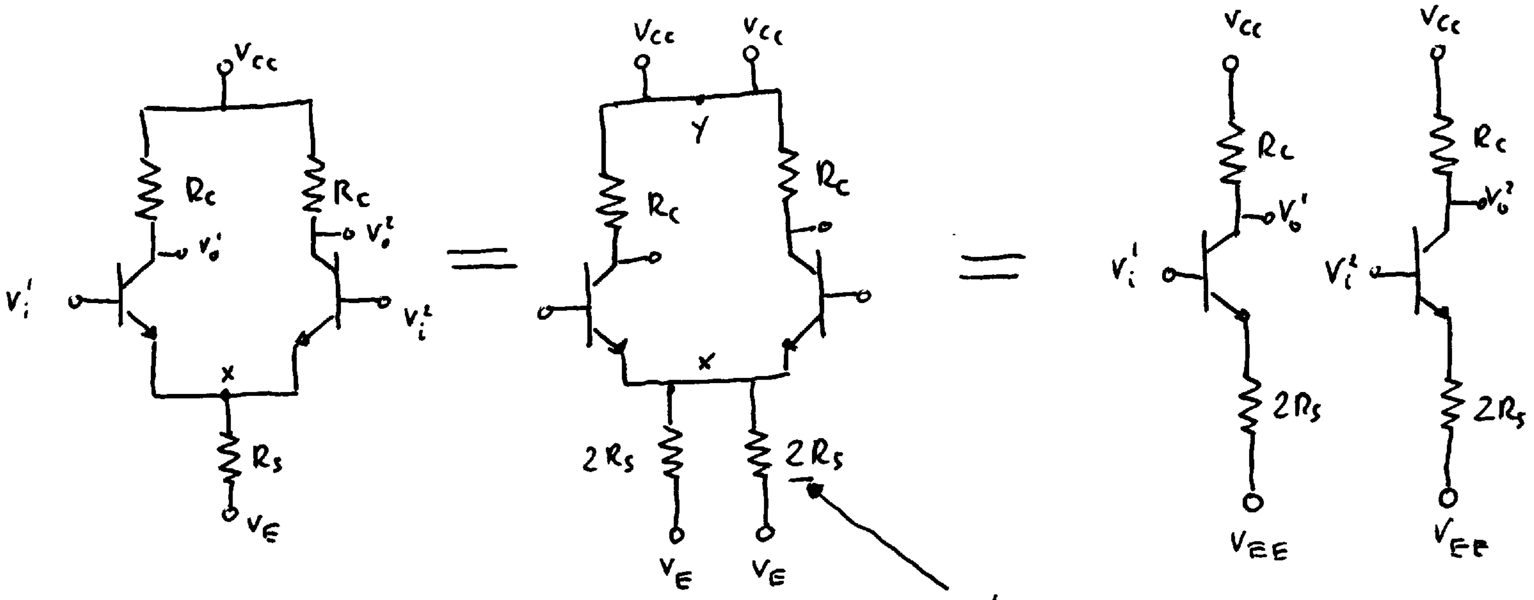
output signal of ~ 1.5 V amplitude. Had we designed the DP to have a bias at 5 V, the maximum output signal would have been 5 V.

Options : increase R_c (to 10.8 kΩ) or decrease R_s (to 3.1 kΩ)

But ... let's continue with this poor DP

Common Mode Gain (A_{cm}).

since at all points on the left the voltages and currents are exactly the same as their counterpoints on the right, we can use a trick of symmetry:



At x,Y no current flows (because of symmetry) → we can cut this

note factor 2

To calculate $A_{cm} = \dfrac{V_o^1}{V_i^1}$ is no problem. This is a normal common emittor amplifier:

$$\boxed{A_{cm} = -\frac{R_c}{r_e + 2R_s}} = -\frac{3.3\,k\Omega}{56\,\Omega + 2\times 10\,k\Omega} = -0.165$$

Alternative way of analyzing (for ~~those~~ who don't like to use tricks and laws of physics) similar to p.5 of ch.0

$$A_{cm} = \left.\frac{\partial V_{o1}}{\partial V_{i1}}\right|_{V_{i1}=V_{i2}}$$

$$I_s = I_{E1} + I_{E2} = 2 I_{E1}$$
$$V_x = V_{E1} = V_{E2}$$

$$\frac{\partial I_{B1}}{\partial V_{B1}} = \frac{1}{\beta+1}\,\frac{\partial I_{E1}}{\partial (V_{E1}+0.7V)} = \frac{1}{\beta+1}\,\frac{\frac{1}{2}\partial I_s}{\partial V_x} = \frac{1}{2(\beta+1)}\cdot\frac{1}{R_s}$$

V_{BE} constant ⇒ $r_e = 0$

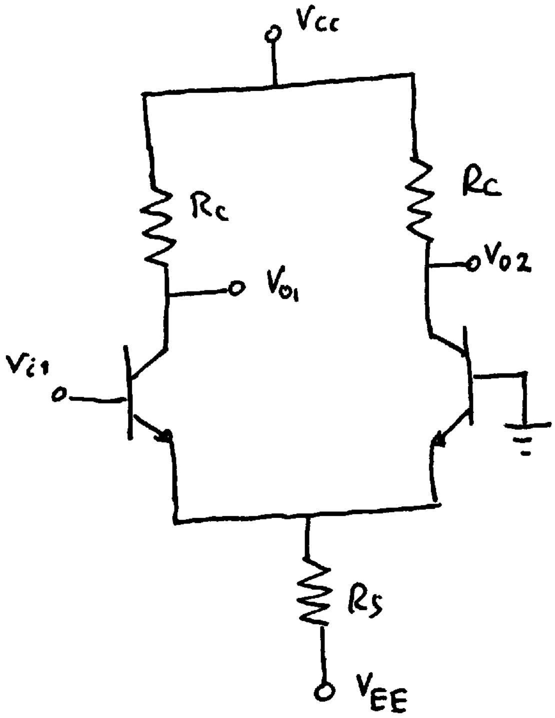
V_{BE} not constant ⟶ $\dfrac{1}{(\beta+1)}\;\dfrac{1}{r_e + 2R_s}$

$$\frac{\partial V_o^1}{\partial V_i^1} = \frac{\partial V_{o1}}{\partial I_{C1}}\cdot\frac{\partial I_{C1}}{\partial I_{B1}}\cdot\frac{\partial I_{B1}}{\partial V_{i1}} = -R_c\cdot\beta\cdot\frac{1}{(\beta+1)}\,\frac{1}{r_e+2R_s} = -\frac{R_c}{r_e+2R_s}$$

q.e.d.

Differential Mode Gain (A_{dm})

$$A_{dm} = \frac{V_{o2} - V_{o1}}{V_{i2} - V_{i1}}$$. For our analysis we will connect the second input to ground ($V_{i2} = 0$)

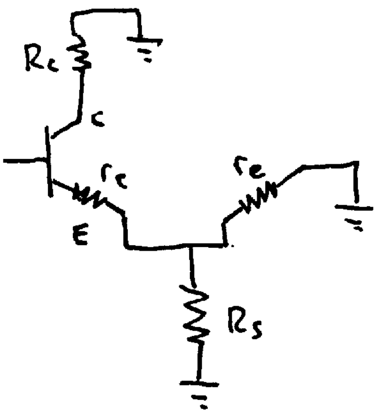


⟸ DP in DM

$$A_{dm} = \frac{V_{o1}}{V_{i1}} - \frac{V_{o2}}{V_{i1}}$$

single-ended common-mode gain

To calculate the gain $\frac{V_{o1}}{V_i}$ is easy: All resistance at collector divided by all resistance at emittor.

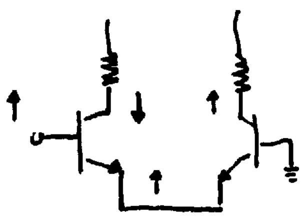


$$\frac{V_{o1}}{V_{i1}} = -\frac{R_c}{r_e + r_e // R_S} \sim -\frac{R_c}{2r_e}$$

This is the single-ended DM gain A^{se}_{dm}

The other output has the same gain, but with positive sign

$$\frac{V_{o2}}{V_{i1}} = +\frac{R_c}{2r_e}$$ why?



V_{i1} increases → V_{BE1} increases → I_{c1} increases → V_{o1} decreases → negative sign

V_{i1} increases → V_{E1} increases (albeit a little less) → V_{BE2} decreases → I_{B2} decreases → I_{c2} decreases → V_{o2} increases.

Another way to understand this:

Because we have a <u>current source</u>, the current is constant $I_{E1} + I_{E2} = \text{constant} \Rightarrow i_{e1} = -i_{e2} \Rightarrow$

$$\left.\begin{array}{l} i_{c2} = -i_{c1} \\ v_{o2} = -R\, i_{c2} \,,\ v_{o1} = -R\, i_{c1} \end{array}\right\} \ v_{o2} = -v_{o1}$$

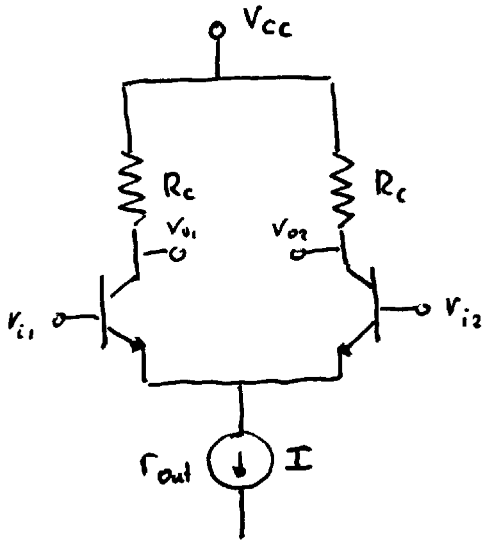
$$\boxed{A_{dm}} = \frac{v_{o1}}{v_{i1}} - \frac{v_{o2}}{v_{i1}} = -\frac{R_c}{2r_e} - \frac{R_c}{2r_e} = \boxed{-\frac{R_c}{r_e}} = -58.9$$

and

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{-R_c/r_e}{-R_c/(r_e+2R_s)}\right| = \frac{2R_s + r_e}{r_e} = 358$$

To improve the CMRR we should increase R_s in some way. The best way to do it is to replace R_s with a current source. Theoretically $r_{out} = \infty$ (ideal current source)

Model 2



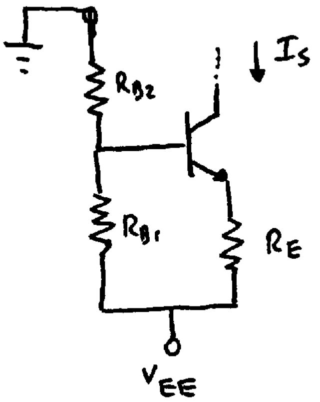
$$A_{cm} = -\frac{R_c}{r_e + 2r_{out}} = 0 \quad \text{(ideal)}$$

$$A_{dm} = -\frac{R_c}{r_e}$$

$$CMRR = \frac{2r_{out} + r_e}{r_e} = \infty \quad \text{(ideal)}$$

The problem now boils down to: how to make a current source with maximum r_{out}. The "current source" of model 1 is rather poor and has an r_{out} of R_s ($\sim k\Omega$'s)

Current source:



r_i of transistor $\gg R_{B1}, R_{B2} \Rightarrow$

$$V_B = \frac{R_{B2}}{R_{B1}+R_{B2}} \cdot V_{EE}$$

$$V_E = V_B - 0.7$$

$$I_E = (V_E - V_{EE})/R_E = \left[\frac{R_{B2}}{R_{B1}+R_{B2}} \cdot V_{EE} - 0.7 - V_{EE}\right]/$$

"Easy calculation" will show that the output resistance of this current source is

$$r_{out} = r_o + \left[(r_\pi + R_B) // R_E\right] + \frac{r_o \beta R_E}{r_\pi + R_B + R_E}$$

with r_o equal to output resistance of transistor $\left(\frac{V_A}{I_E}\right)$

$$r_\pi = (\beta+1)\, r_e$$

$$R_B = R_{B1} // R_{B2}$$

Example : $R_{B1} = 10\ k\Omega$, $R_{B2} = 10\ k\Omega$, $R_E = 4.7\ k\Omega$

$r_o = 200\ k\Omega$, $\beta = 100$, $r_\pi = 2\ k\Omega$, $V_{EE} = -10\ V$

$V_B = -5\ V$, $V_E = -5.7\ V$, $I_E = 0.91\ mA \Rightarrow \boxed{I_S = 0.91\ mA}$

$$r_{out} = 200\ k\Omega + \left[(2\ k\Omega + 5\ k\Omega) // 4.7\ k\Omega\right] + \frac{200\ k\Omega \times 100 \times 4.7\ k\Omega}{2\ k\Omega + 5\ k\Omega + 4.7\ k\Omega}$$

$$= 200\ k\Omega + 2.8\ k\Omega + 100 \times 80\ k\Omega = \boxed{8.2\ M\Omega}$$

OUR DP WITH THIS SOURCE:

$A_{cm} = -2.0\ 10^{-4}$

$A_{dm} = -58.9$ (unchanged)

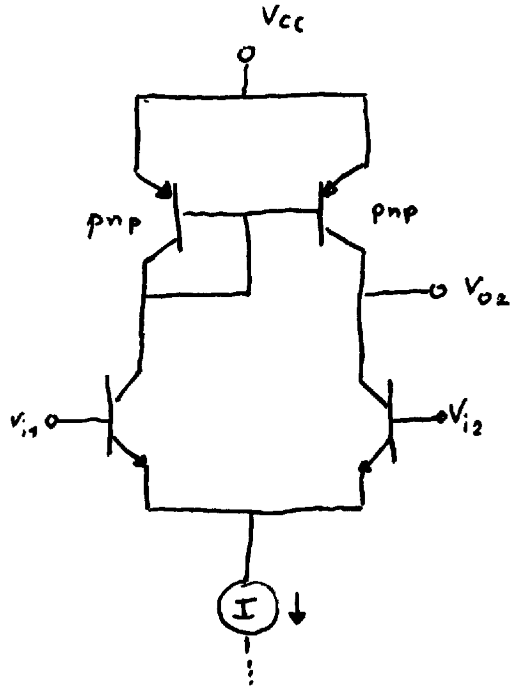
$CMRR = \left|\frac{-58.9}{-2.0\ 10^{-4}}\right| = 2.9\ 10^{5}$ (800 times better!)

To further increase the CMRR we can try to increase the differential gain A_{dm}. Remember

$$A_{dm} = -R_c / r_e$$

So we should increase R_c. This can be done by replacing the collector resistances by (pnp) transistors, whose output resistance is r_o (100's kΩ):

Differential pair with active load



$$A_{cm} = -\frac{r_o}{r_e + 2\, r_{out}}$$

r_o of pnp trans. 200 kΩ

r_{out} of current source 8.2 MΩ

$$A_{dm} = -\frac{r_o}{r_e}$$

$$CMRR = \frac{2\, r_{out} + r_e}{r_e} \quad \text{(unchanged!)}$$

In our example $r_e = 56\ \Omega$:

$$A_{cm} = -\frac{200\ k\Omega}{56\ \Omega + 2 \times 8.2\ M\Omega} = -1.22\ 10^{-2}$$

$$A_{dm} = -\frac{200\ k\Omega}{56\ \Omega} = 3570$$

$$CMRR = 2.9\ 10^{5} \quad \text{unchanged!}$$

The advantage of the active load is not a higher CMRR. So why are all DP's implemented with active load?

The reason is the fact that there are no resistances in the circuit (as we will see, the current source can be made by current mirrors). In integrated circuits it is very difficult to make resistances (where it is easy to make transistors).

One final observation. We used here the single-ended gain / output. Only v_{o2}. In fact, the gain at v_{o1} is much lower. When we connect v_{i2} to ground (differential mode):

$$\left.\begin{aligned} \frac{v_{o1}}{v_{i1}} &= -\frac{r_\pi^p // r_\pi^p}{2r_e^n} \approx -\frac{\beta}{4} \\ \frac{v_{o2}}{v_{i1}} &= \frac{r_o^p}{2r_e^n} \end{aligned}\right\} \quad A_{dm} \approx -\frac{r_o^p}{2r_e}$$

(r_π^p is of pnp, r_e^n of npn tr.)
(r_o^p is of pnp)

common mode ($v_{i1} = v_{i2}$)

$$\left.\begin{aligned} \frac{v_{o1}}{v_{i1}} &= -\frac{r_\pi^p // r_\pi^p}{r_e^n + 2r_o^s} \\ \frac{v_{o2}}{v_{i1}} &= -\frac{r_o^p}{r_e^n + 2r_o^s} \end{aligned}\right\} \quad A_{cm} \approx \frac{r_o^p}{2r_o^s}$$

(r_o^s is of current source)

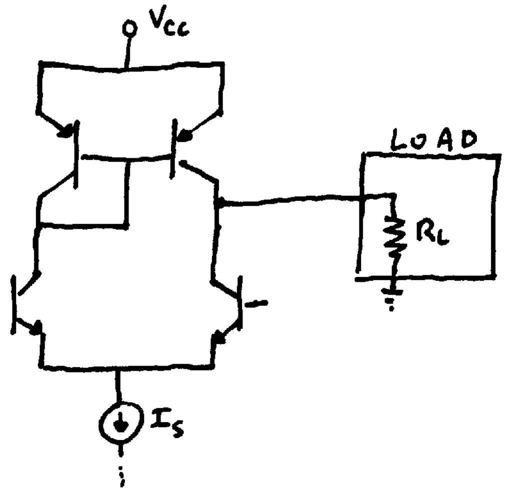
$$CMRR = \frac{r_o^s}{r_e}.$$

(A factor 2 less than of p. 8)

A fact that is not <u>found back</u> in textbooks.

One more observation about the DP with active load:

This is a strange amplifier, because the input signal is voltage, but the output signal is rather current. (ideally, $r_o^o = \infty$). This amplifier only works when connected to something. In that case the gain becomes limited by the load resistance.



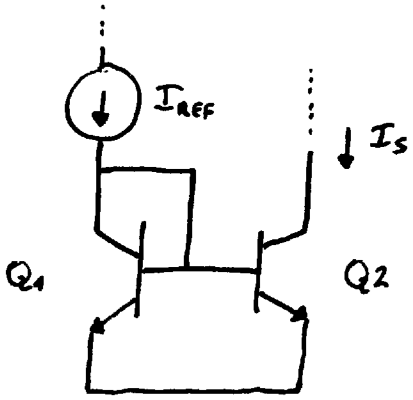
$$A_{cm} = -\frac{R_L}{r_e + 2r_{out}}$$

$$A_{dm} = -\frac{R_L}{r_e}$$

$$CMRR = \frac{2r_{out} + r_e}{r_e} \quad \text{(unchanged)}$$

CURRENT SOURCES

We already used a current source (on p.7) and, as a matter of fact, the active load in the previous amplifier is also a current source, a so-called current mirror. It is called a mirror, because the current in one leg (the right in this case) is equal to the current in the other leg (left in this case). The reason why this is so is easy to see. Consider the next circuit of a current mirror:



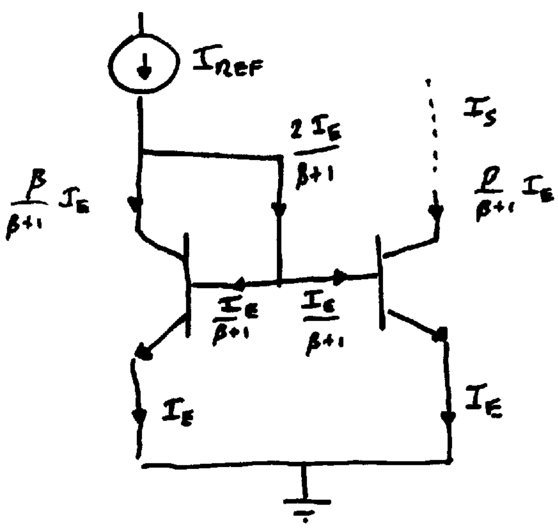
I_S is equal to I_{REF} ! why

Imagine I_{REF} is from ideal current source. This current is 'pushed' through transistor Q1. ("at all cost"). Therefore, a voltage drop V_{BE1} is induced at the base-emittor junction. According to Ebers-Moll (p.2 ch.0) for a diode $I_{E1} = I_{REF} = I_0 \exp[V_{BE}/V_T]$

$$\Rightarrow \quad V_{BE1} = V_T \ln(I_{REF}/I_0)$$

The voltage drop at the right transistor is equal (same base and same emittor voltages). $V_{BE2} = V_{BE1}$. Therefore, $I_{C2} = I_{C1} = I_{REF}$. (using Ebers-Moll).

Here we have assumed an infinite β. For finite β we can see that a little bit of the current I_{REF} is lost for biasing the transistors.



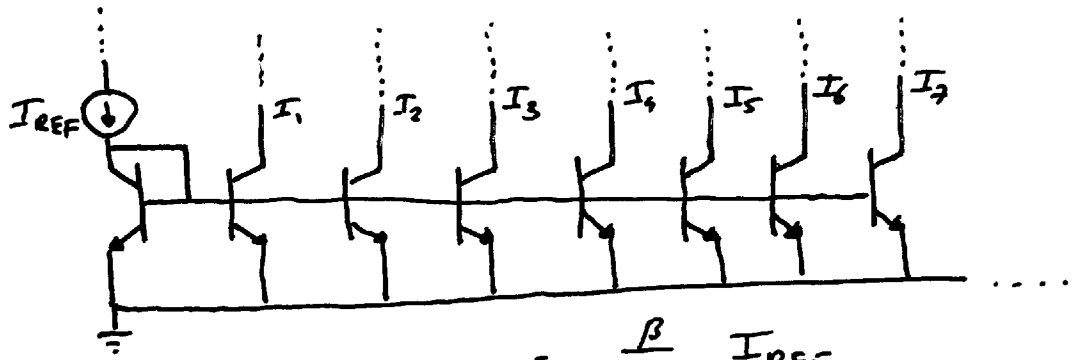
$$\leftarrow \quad I_S = \frac{\beta}{\beta+1} I_E$$

$$I_{REF} = \frac{\beta}{\beta+1} I_E + \frac{2}{\beta+1} I_E$$

$$\Rightarrow I_S = I_{REF}\left(\frac{\beta}{\beta+2}\right)$$

We started with a current source (I_{REF}) and ended with a current source (I_S). Where is the advantage ?

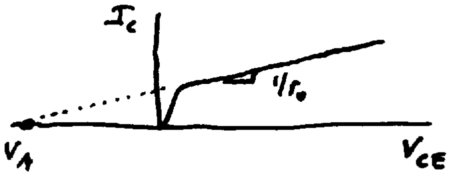
Multi - current - source



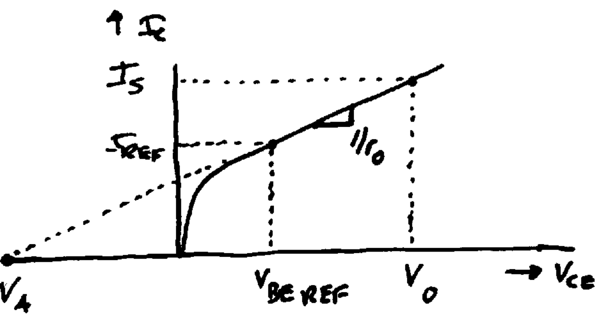
$$I_1 = I_2 = \cdots = \frac{\beta}{\beta+1+n} I_{REF}$$

The output resistance of a current mirror is r_o ($\sim V_A/I_s$)

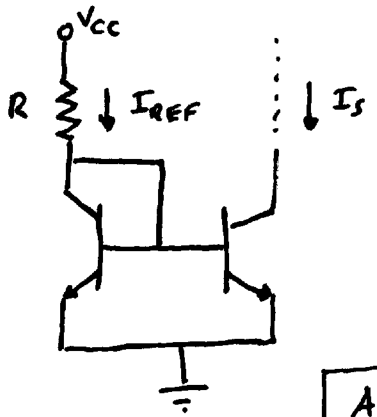
This causes another non-linearity. Since the current I_{CS} depends on the collector voltage, the source current depends on the output voltage, and is not necessarily equal to I_{REF} ($V_{CREF} = V_{BREF} \neq V_{C1}$).



$$I_s = I_{REF} + (V_o - V_{BE,REF})/r_o = I_{REF}\left(1 + \frac{V_o - V_{BE}}{V_A}\right)$$



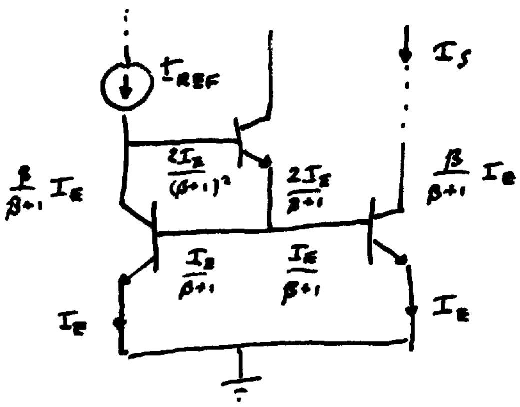
Other current sources / mirrors :



- $I_S = \dfrac{V_{cc} - 0.7}{R}$
- $r_{out} = r_o$

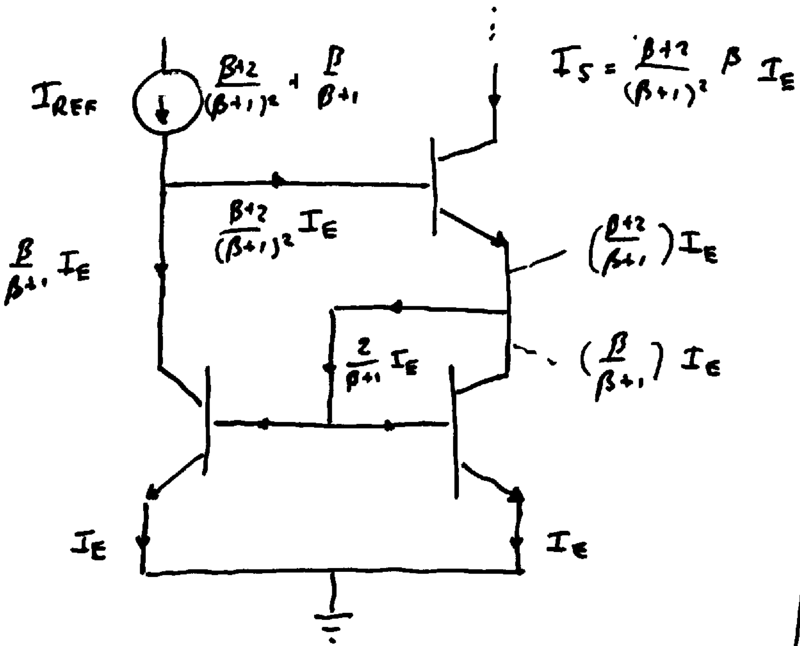
(suffers from "β" and "V_o" effects)

$\boxed{\text{ADDING R TURNS A MIRROR INTO A SOURCE!}}$



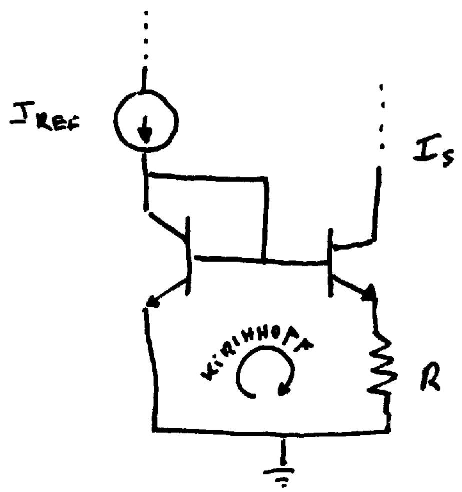
- $I_S \approx I_{REF}$
- $r_{out} = r_o$
- doesn't suffer (so much) from "β" effect (only $2I_E/(\beta+1)^2$ is lost)

WILSON CURRENT MIRROR



- $I_S = I_{REF} \left(\dfrac{1}{1 + 1/(\beta^2 + 2\beta)} \right)$
- reduced "β" effect
- $r_{out} \approx \dfrac{\beta r_o}{2}$

WIDLAR CURRENT SOURCE



$$V_{BEREF} = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$V_{BES} = V_T \ln\left(\frac{I_S}{I_0}\right)$$

Kirchoff : $V_{BEREF} - V_{BES} - I_S R = 0$

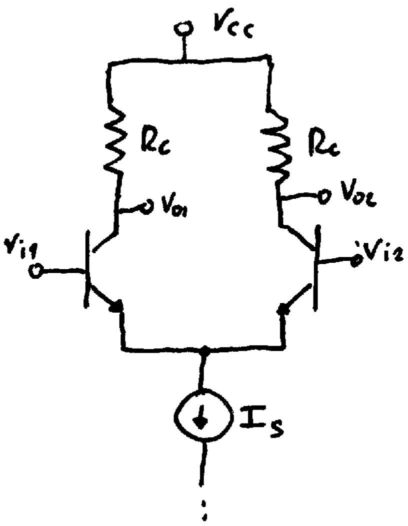
$$I_S R = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

- R very small (still possible with integr. circuits)

  $I_{REF} \approx I_S$

LARGE - SIGNAL ANALYSIS OF D.P.

Until now it was assumed that we worked in the linear region of the D.P. That is $V_o \sim V_i$. Up to what point was this justified?



Using Ebers - Moll

$V_{BE1} = V_{i1} - V_E$

$V_{BE2} = V_{i2} - V_E$

$$I_{E1} = I_0 \exp\left(V_{BE1}/V_T\right)$$

$$I_{E2} = I_0 \exp\left(V_{BE2}/V_T\right)$$

$$\Longrightarrow \frac{I_{E1}}{I_{E2}} = \exp\left(\frac{V_{i1} - V_{i2}}{V_T}\right)$$

also : $I_{E1} + I_{E2} = I_S$

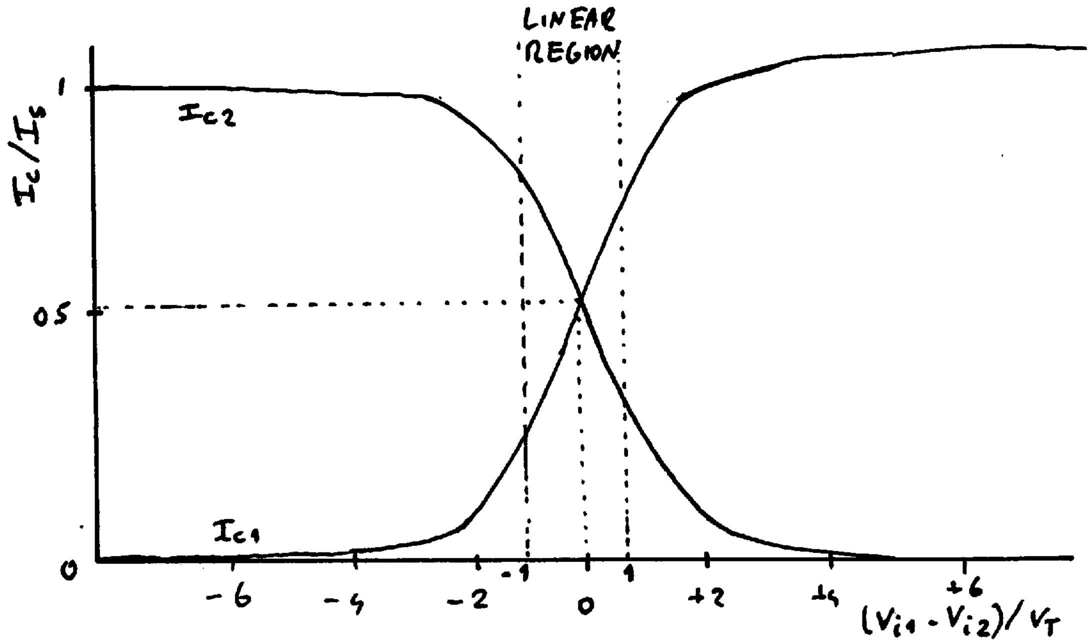
$\Longrightarrow$

$$I_{E1} = \frac{I_S}{1 + \exp\left(\frac{V_{i2} - V_{i1}}{V_T}\right)}, \quad I_{E2} = \frac{I_S}{1 + \exp\left(\frac{V_{i1} - V_{i2}}{V_T}\right)}$$

Assuming $\alpha \simeq 1$ $\quad I_{C1} = I_{E1}$, $I_{C2} = I_{E2}$

The resistances R_C are current-to-voltage translators

$$V_{O1} = V_{CC} - \frac{R_C I_S}{1 + \exp\left(\frac{V_{i2} - V_{i1}}{V_T}\right)} , \quad V_{O2} = V_{CC} - \frac{R_C I_S}{1 + \exp\left(\frac{V_{i1} - V_{i2}}{V_T}\right)}$$



The amplifier is linear oproximately for differential voltages not exceeding V_T !

$\boxed{\text{USE INPUT SIGNALS OF} \sim 20\,mV}$