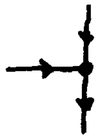
CHAPTER 0

Revision of Electronics 1

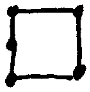
A : Leis de Kirchhoff

$\sum I = 0$



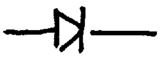
cargas não desaperecem

$\sum V = 0$



voltar ao início : mesma tensão

B :

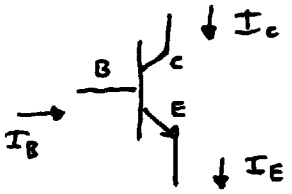


diodo

aberto : $\Delta V \sim 0.7$, $I > 100\,\mu A$

fechado : $I \sim 10^{-14}\,A$ (~ 0)

C : (junction) transistor is current amplifier



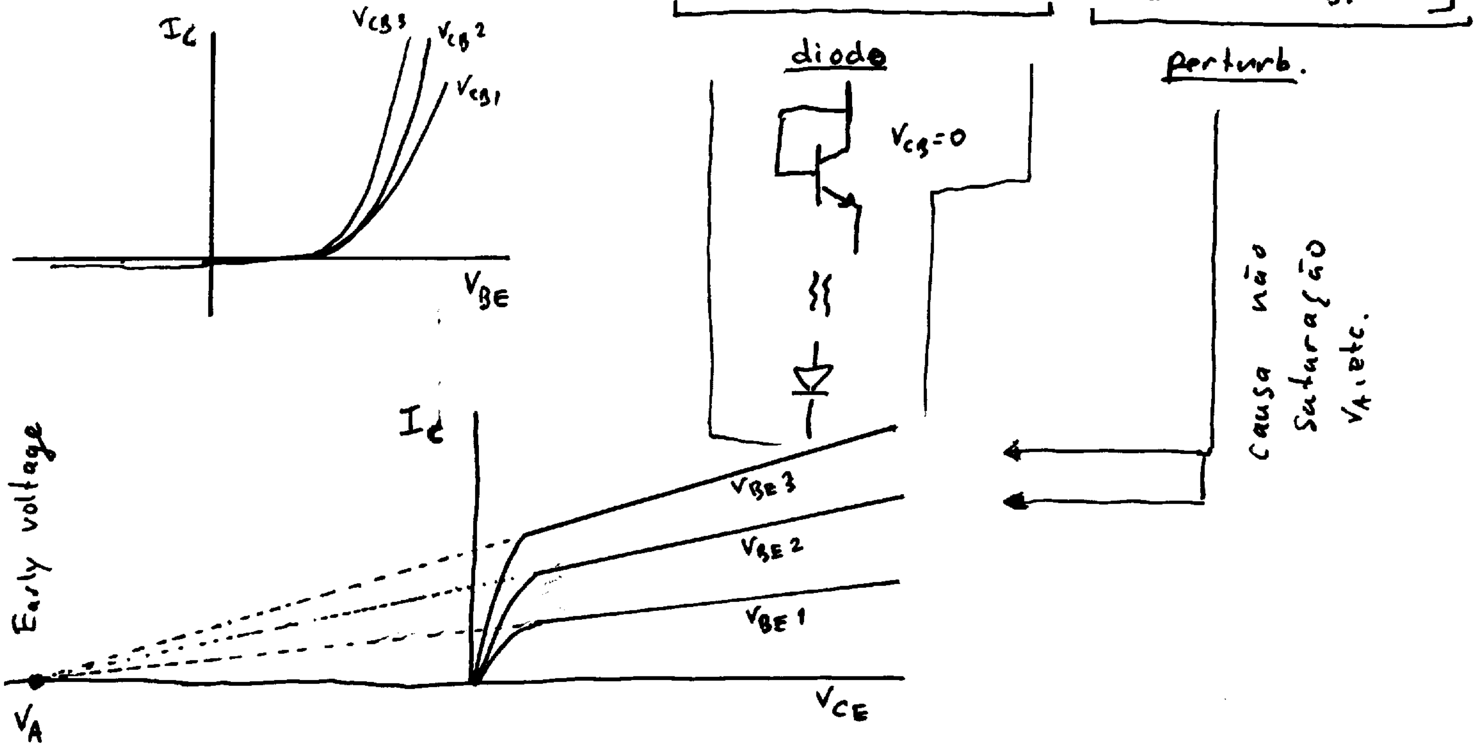
$$\left.\begin{aligned} I_E &= (\beta+1) I_B \\ I_c &= \beta I_B \\ I_c &= \alpha I_E \end{aligned}\right\} \text{(Kirchhoff } I_c + I_B = I_E\text{)}$$

$$\alpha = \frac{\beta}{\beta+1} \approx 1$$

typical : $\beta = 100$, $\alpha = 0.99$

D : Ebers - Moll :

$$I_C = \underbrace{I_0\left[\exp\left(\frac{q}{kT}V_{BE}\right)-1\right]}_{\text{diodo}} + \underbrace{I_1\left[\exp\left(\frac{q}{kT}V_{CB}\right)-1\right]}_{\text{perturb.}}$$



E : simplificação

$$I_E = I_0 \left[\exp\left(\frac{q}{kT} V_{BE}\right) - 1 \right]$$

$I_0 \sim 10^{-14}$ A   reverse-bias-leakage current
$(V_{BE} = -\infty \, , \; I_E = -I_0)$

$$I_E \sim I_0 \exp\left(\frac{q}{kT} V_{BE}\right) = I_0 \exp\left(V_{BE}/V_T\right) \qquad V_T \equiv \frac{kT}{q}$$

corrente típica : 1 mA , T = 300K (V_T = 26 mV)

$V_{BE} = V_T \ln\left(I_E/I_0\right)$:

0.60 V ↔ 100 μA
0.66 V ↔ 1 mA
0.72 V ↔ 10 mA
0.78 V ↔ 100 mA

$$\boxed{\text{Transístor aberto} \longleftrightarrow V_{BE} = 0.7 \text{ V}}$$

F : Resistência de entrada

$$\boxed{r_i \equiv 1 \Big/ \frac{\partial I_i}{\partial V_i}}$$

dinâmico

example : $V_i = V_{BE}$
$I_i = I_B = \frac{1}{(\beta+1)} I_E$

$$\frac{\partial I_i}{\partial V_i} = \frac{1}{(\beta+1)} \frac{\partial I_E}{\partial V_{BE}} = \frac{I_0}{(\beta+1) V_T} \exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_E}{(\beta+1) V_T}$$

$$r_i = (\beta+1) \frac{V_T}{I_E} = (\beta+1) r_e = r_\pi \qquad 2.6 \text{ k}\Omega$$

$\Uparrow$

$$\boxed{R_i \equiv 1 \Big/ \frac{I_i}{V_i}}$$

estático

example : $R_i = 1 \Big/ \frac{I_E}{(\beta+1) V_{BE}}$ 70 kΩ

$\Uparrow$

example com $\beta = 100$, $V_{BE} = 0.7$, $1 \text{mA} = I_E$

note : $g_m \equiv \frac{\partial I_{out}}{\partial V_i} = \frac{\beta \, \partial I_i}{\partial v_i} = \frac{\beta}{\beta+1} \cdot \frac{1}{r_e}$

---

r_e is a parameter that describes the dynamic response to a signal input (δV_i). It is not a real resistance!

$v_i = \delta V_i$

$$r_e \equiv V_T / I_E$$

(depends on bias I_E and temperature)

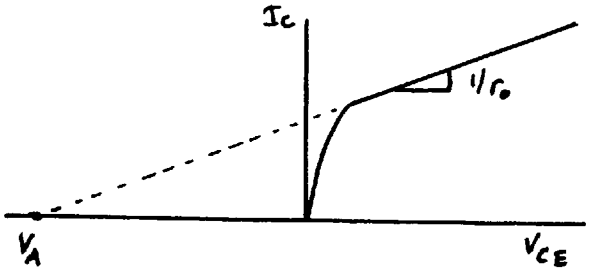
G : Resistência de saida

$$\boxed{r_o \equiv 1 / \frac{\partial I_o}{\partial V_o}}$$

exemplo : transistor

$I_o = I_c$

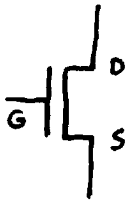
$V_o = V_c = V_{CE}$



$$\frac{\partial I_o}{\partial V_o} = \frac{\partial I_c}{\partial V_{CE}} = \frac{I_c}{V_{CE} + V_A}$$

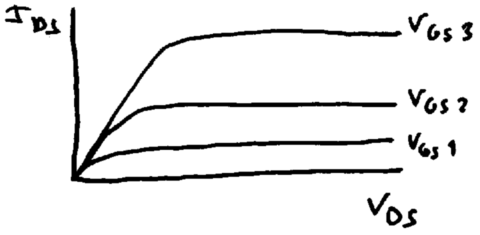
$$r_o \sim \frac{V_A}{I_c} \text{ , ex } \frac{200\ V}{1mA} = 200\ k\Omega$$

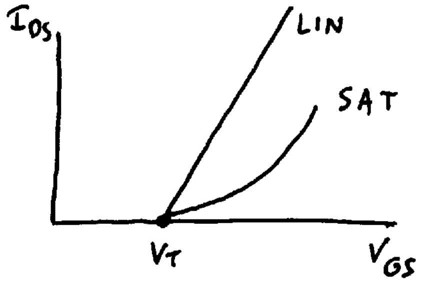
H : Field - Effect Transistor



LIN : $I_{ds} = C_{ox}\,\mu\,\frac{W}{L}\,(V_{GS} - V_T)\,V_{DS}$

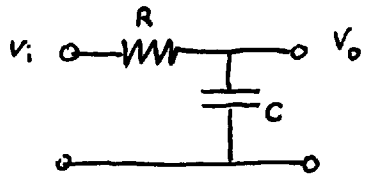
SAT : $I_{ds} = \frac{1}{2}\,C_{ox}\,\mu\,\frac{W}{L}\,(V_{GS} - V_T)^2$





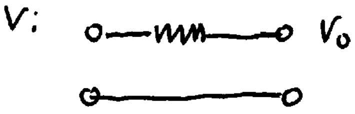
$$r_o = 1 / \frac{\partial I_{DS}}{\partial V_{DS}} = \infty \qquad r_i = 1 / \frac{\partial I_G}{\partial V_{GS}} = \infty \quad (I_{GS} = 0)$$

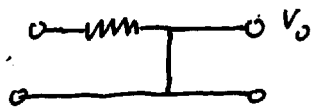
I : Filtros



"em altas frequências um C é um curto-circuito"
"em baixas frequências um C é um circuito-aberto"

LPF (low-pass filter)



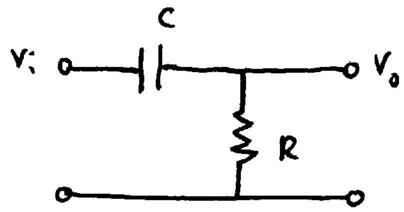


baixas - freq. ⟷ altos - freq.

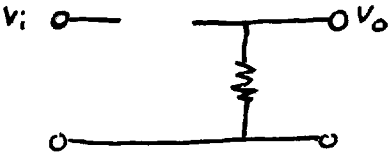
$V_o = V_i$ $\qquad f_c \qquad$ $V_o = 0$

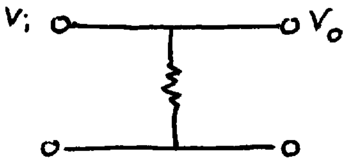
ponto de comutação : $f_c = \frac{1}{2\pi RC}$

(p 902 Sedra)



HPF (high-pass filter)



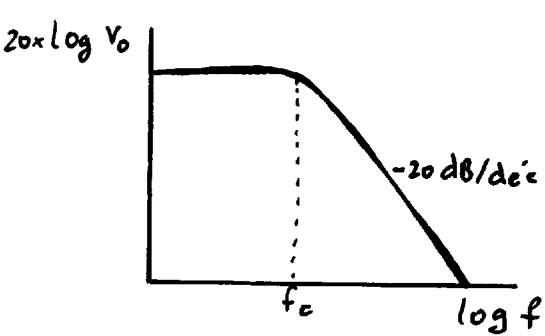


baixas - freq. ⟷ altas - freq

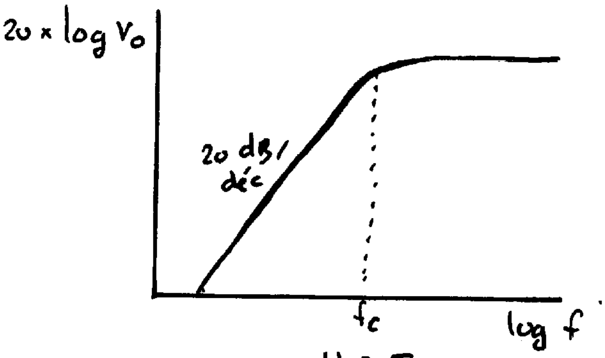
$V_0 = 0$ f_c $V_0 = V_i$

ponto de comutação $f_c = \frac{1}{2\pi RC}$

J: Bode plots ($20\times\log V_0$ - $\log f$)



LPF

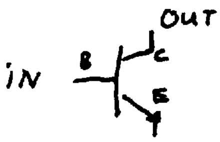


HPF

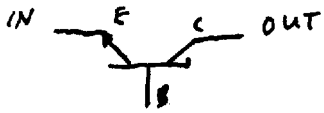
em f_c : $V_0 = \frac{1}{\sqrt{2}} V_0^{max}$ ($P_0 = \frac{1}{2} P_0^{max}$)

$\log V_0 = -3 dB$ ($20\times{}^{10}\log \frac{1}{\sqrt{2}} = -3$ dB)

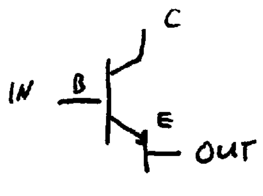
M: SIMPLE CIRCUITS



COMMON EMITTOR

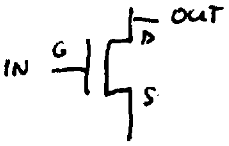


COMMON BASE

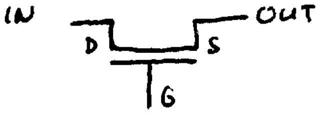


COMMON COLLECTOR

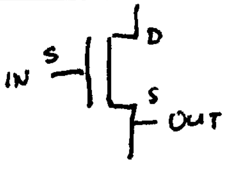
Small - signal amplifiers are named after the terminal that has neither input nor output



COMMON SOURCE



COMMON GATE



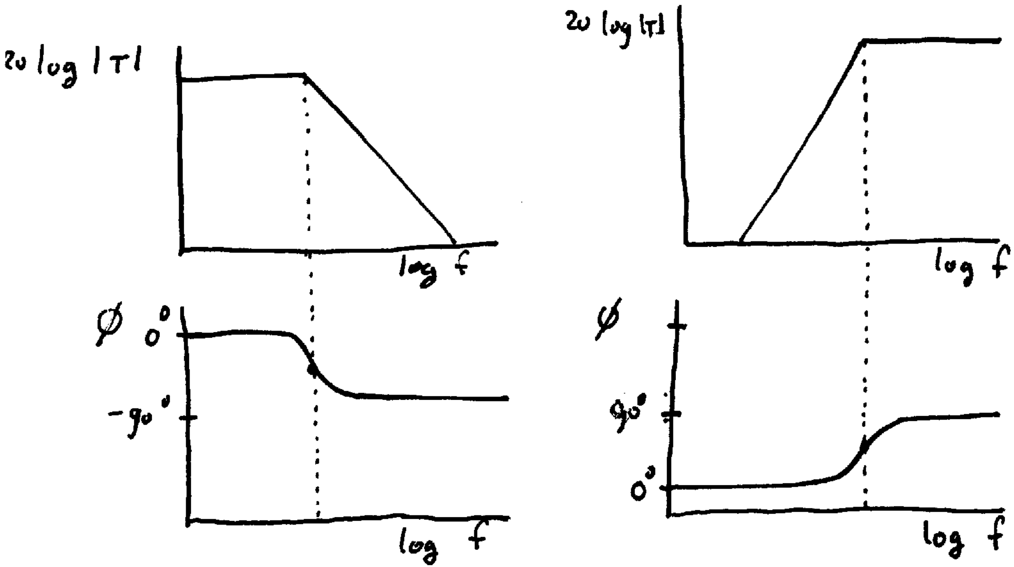
COMMON DRAIN

K: Transfer functions

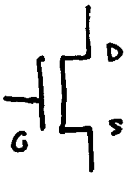
$$T(f) \equiv V_o(f)/V_i(f) \quad \text{or} \quad T(s) = V_o/V_i$$
$$s = j\omega$$

	LPF	HPF	
$T(s)$	$\frac{1}{1+s/\omega_o}$	$\frac{s}{s+\omega_o}$	
$T(\omega)$	$\frac{1}{1+j\omega/\omega_o}$	$\frac{1}{1-j\,\omega_o/\omega}$	
$\lvert T(\omega)\rvert$	$\frac{1}{\sqrt{1+(\omega/\omega_o)^2}}$	$\frac{1}{\sqrt{1+(\omega_o/\omega)^2}}$	
$\omega = 0$	1	0	
$\omega = \infty$	0	1	
$\omega = \omega_o$	$\lvert T\rvert = \frac{1}{\sqrt{2}},\ \phi = -45^\circ$	$\lvert T\rvert = \frac{1}{\sqrt{2}},\ \phi = +45^\circ$	(-3dB)
$\omega << \omega_o$	1	$\sim \omega$ (20 dB/dec)	
$\omega >> \omega_o$	$\sim 1/\omega$ (-20 dB/dec)	1	
$\omega_o =$	$\frac{1}{RC}$	$\frac{1}{RC}$	

Bode Plots

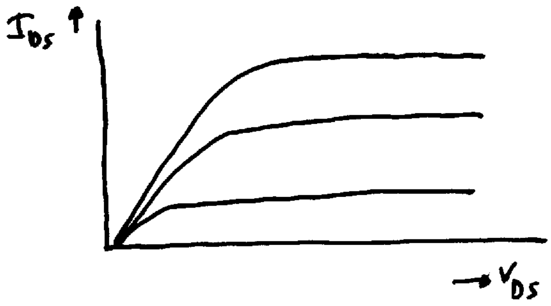


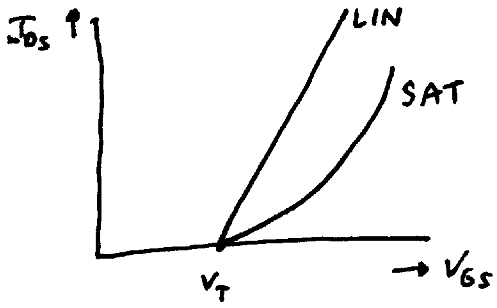
H: Field - Effect Transistor



LIN : $I_{ds} = C_{ox}\,\mu \frac{W}{L}\,(V_{GS}-V_T)\,V_{DS} = \lambda\,(V_{GS}-V_T)\,V_{DS}$

SAT : $I_{ds} = \frac{1}{2}\,C_{ox}\,\mu \frac{W}{L}\,(V_{GS}-V_T)^2 = \frac{1}{2}\,\lambda\,(V_{GS}-V_T)^2$





$$r_0 = 1 \Big/ \frac{\partial I_{DS}}{\partial V_{DS}}\Bigg|_{V_{GS}=\text{const}} = \infty \quad , \quad r_i = 1 \Big/ \frac{\partial I_G}{\partial V_{GS}}\Bigg|_{V_{DS}=\text{const}} = \infty \quad (I_{GS}=0)$$

(no leakage current or polarization current !)

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}}\Bigg|_{V_{DS}=\text{const.}} = \lambda\,V_{DS} \quad (\text{LIN})$$

$$= \lambda\,V_{GS} \quad (\text{SAT})$$

L: components analyzed

$V = I.R$

→ A resistance is a linear element that translates current to voltage

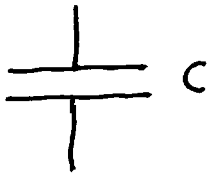
$I \rightarrow V$

→ A resistance defines a current

$V \rightarrow I$

$\partial V = R \cdot \partial I$

$(V = R \cdot i)$



$C = Q/V$

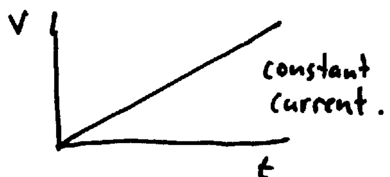
→ A capacitor is a charge storage

"how much charge stored per volt" $V \rightarrow Q$

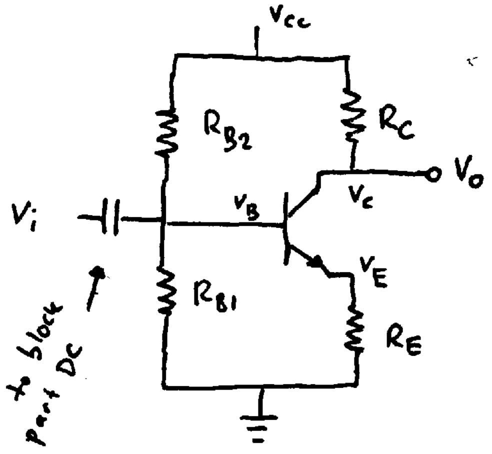
→ is "integrator"

$Q \rightarrow V$

$\int I dt \rightarrow V$



Example common-emitter amplifier PART 1B



$\beta = 100$

$R_{B1} = R_{B2} = 10\ k\Omega$

$R_E = 3.3\ k\Omega$

$R_c = 3.3\ k\Omega$

$V_{CC} = +10\ V$

POLARIZATION / BIAS

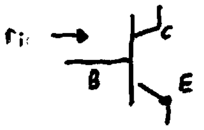
1) V_B ?

Assumption r_i at transistor $\gg 10\ K$

why?

$r_i = r_\pi + R_E$? No!

$r_i = r_\pi + \underline{(\beta+1)} R_E$!



remember : $r_i \equiv 1 / \frac{\partial I_B}{\partial V_B}$

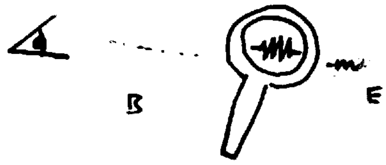
$$\frac{\partial I_B}{\partial V_B} = \frac{\frac{1}{\beta+1}\,\partial I_E}{\partial\,(V_E + \underset{\uparrow}{0.7}\ V)} = \frac{1}{\beta+1}\,\frac{\partial I_E}{\partial V_E} = \frac{1}{(\beta+1)\,R_E}$$

implies $r_\pi = 0$

$r_i = (\beta+1)\,R_E$

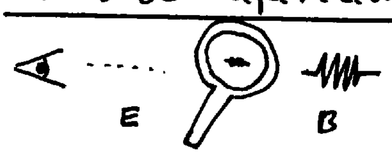
total (including r_π) : $r_i = \underset{\substack{\uparrow \\ \text{order} \\ 1\ k\Omega}}{r_\pi} + \underset{\substack{\uparrow \\ \text{order} \\ 100\ k\Omega}}{(\beta+1)\,R_E}$

O transistor funcione como uma lente. Vista da base, as resistências (e outros componentes) aperecem ampliadas.



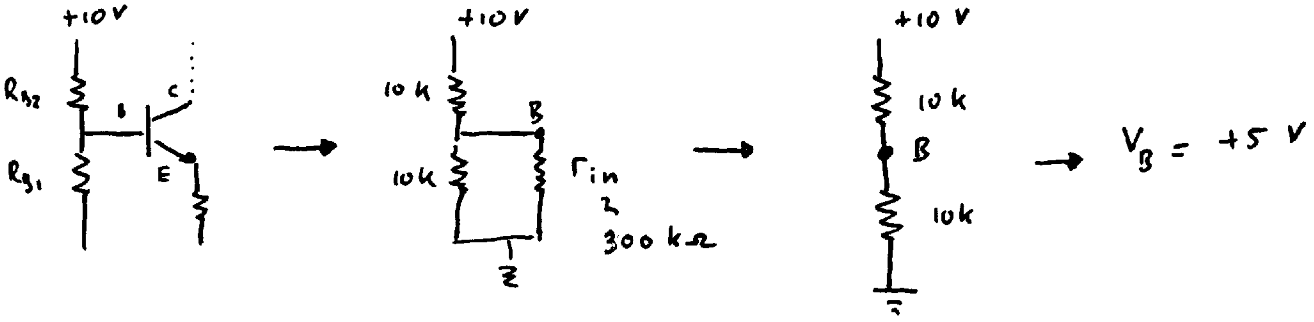
$(\beta+1)R_E \longleftarrow R_E$

Vista do emisor, os componentes na base aperecem reduzidos



$R_B \longrightarrow \frac{R_B}{\beta+1}$

$r_{in} >> R_B$:



$$V_B = \frac{R_{B1}}{R_{B1} + R_{B2}} \cdot V_{cc} = \frac{10\ k\Omega}{10\ k\Omega + 10\ k\Omega} \cdot 10\ V = 5\ V$$

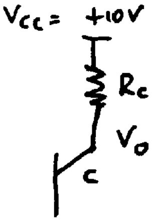
$V_E = V_B - 0.7$ V (in case transistor open. check at end!)

$= 4.3$ V

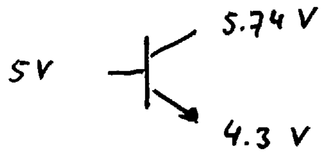
$I_E = V_E / R_E = 1.30$ mA

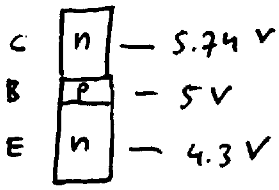
$I_B = I_E/(\beta+1) = 12.9\ \mu A$

$I_C = \alpha I_E = 1.29$ mA



$V_o = V_{cc} - R_c \cdot I_c = 10 - 3.3\ k\Omega \times 1.29\ mA = 5.74$ V





A properly working transistor has the C-B biased reverse and the B-E junction in forward . Forward bias : n = negative , p = positive. Thus, the above transistor is working correct.

$$r_e = \frac{V_T}{I_E} = \frac{26\ mV}{1.30\ mA} = 20\ \Omega$$

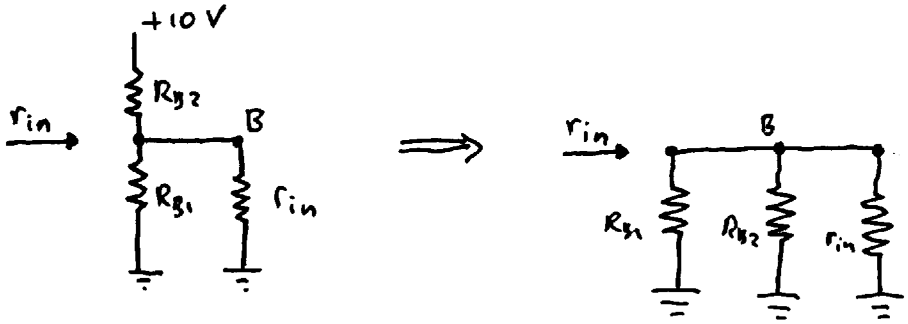
$$r_\pi = (\beta+1)\, r_e = (100+1)\, 20\,\Omega = 2\ k\Omega$$

$$r_{in} = r_\pi + (\beta+1) R_E = 335\ k\Omega$$

Total dynamic input resistance of amplifier:

> For AC signals : any DC power supply point is equal to ⏚ (and current source = open)

(because v (= δv) ≡ 0 !



$r_{in} = R_{B1} // R_{B2} // r_{in}$

$= 10\,k\Omega // 10\,k\Omega // 335\,k\Omega$

$\sim 5\,k\Omega$

AC signal amplification:

$$r_i = r_\pi + (\beta+1) R_E$$

$$r_i \equiv 1 \Big/ \frac{\partial I_B}{\partial V_B} \Rightarrow \partial I_B = \frac{\partial V_B}{r_i} = \frac{\partial V_B}{r_\pi + (\beta+1) R_E}$$

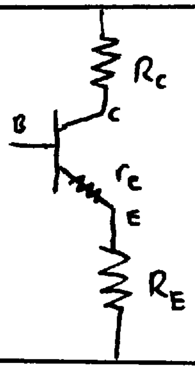
$$I_c = \beta I_B \Rightarrow \partial I_c = \beta\, \partial I_B = \frac{\beta\, \partial V_B}{r_\pi + (\beta+1) R_E}$$

$$V_o = V_{cc} - R_c I_c \Rightarrow$$

$$\partial V_o = -R_c \cdot \partial I_c = -\frac{\beta R_c}{r_\pi + (\beta+1) R_E} \cdot \partial V_B$$

minúsculas ↘ maiúsculas ↘

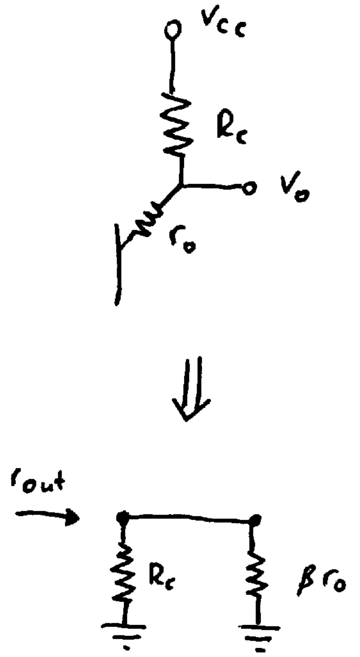
$$A = \frac{v_o}{v_B} = \frac{\partial V_o}{\partial V_B} = -\frac{R_c}{r_e + R_E} \qquad \left(r_\pi = (\beta+1) r_e \;,\; \beta \approx (\beta+1) \right)$$



> The gain of a common-emittor amplifier is <u>all resistance at collector</u> divided by <u>all resistance at emittor</u> with minus sign

$$A = -\frac{3300}{20 + 3300} \sim -1$$

output resistance of amplifier



r_o is resistance of transistor (with E connected to ground !!)

$r_o \approx 200\ k\Omega$ (p. 3)

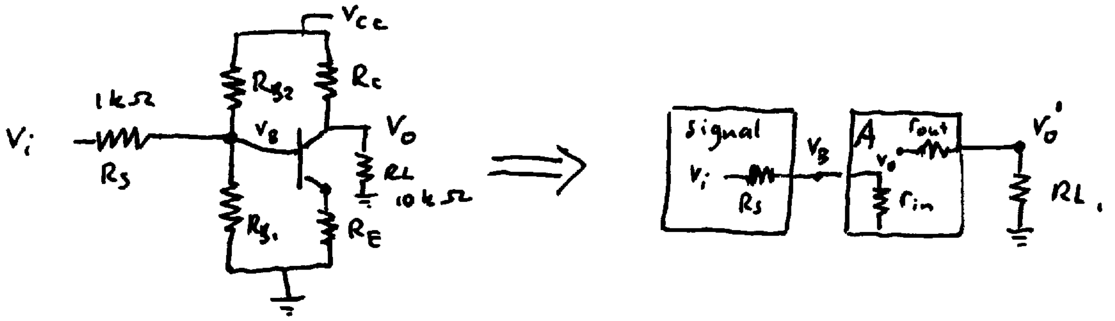
$(r_o >> R_c)$

r_{out} of transistor in Com. Em. is MUCH larger than r_o (something like $\beta \times r_o$) (see p. 7 of ch. 1)

$r_{out} \sim R_c = 3.3\ k\Omega$

Combining amplifier stages

imagine signal source has 1 kΩ output resistance and a 10 kΩ load resistance is at output what will be the total gain of circuit ?



$R_s = 1\,k\Omega$, $r_{in} = 5\,k\Omega$ (p. 7) , $R_L = 10\ k\Omega$

$A = -1$ (p. 7) , $r_{out} = 3.3\ k\Omega$ (p. 8)

$$A_{tot} = \frac{V_o'}{V_i} = \frac{V_o'}{V_o} \cdot \frac{V_o}{V_B} \cdot \frac{V_B}{V_i}$$

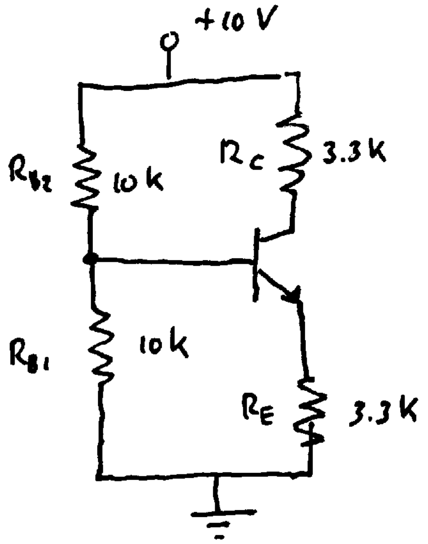
$$\frac{V_o'}{V_o} = \frac{R_L}{R_L + r_{out}} \quad , \quad \frac{V_o}{V_B} = A \quad , \quad \frac{V_B}{V_i} = \frac{r_{in}}{r_{in} + R_s}$$

$$= 0.429 \qquad\qquad = -1 \qquad\qquad = 0.833$$

$$A_{tot} = 0.429 \times -1 \times 0.833 = -0.357$$

DC power consumption of amplifier

$P = I^2R$ or V^2/R or VI



R_{B1} : $V = 5V$, $R = 10\,k \Rightarrow P = 2.5$ mW

R_{B2} : $V = 5V$, $R = 10\,k \Rightarrow P = 2.5$ mW

R_E : $V = 4.3\,V$, $R = 3.3\,k \Rightarrow P = 5.6$ mW

R_C : $V = 4.3\,V$, $R = 3.3\,k \Rightarrow P = 5.6$ mW

* trans : $V = 0.7\,V$, $I = 1.29\,mA \Rightarrow P = 0.90$ mW

total $P = 17.1$ mW

Note : there is power loss at every part where we have a voltage drop and a current simultaneously. You can also say : where there is a current and a resistance. The heat generated in the transistor is not much in this case, but other amplifiers will lose a lot of power in the transistors and have to be cooled.

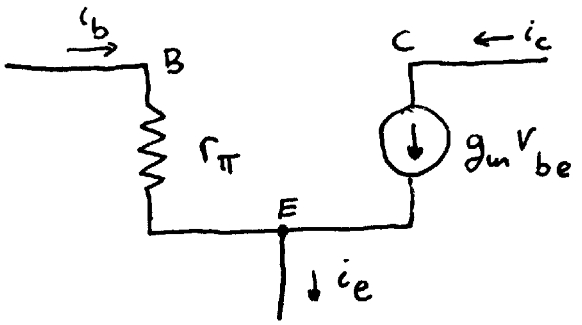
* : note that we cannot use r_e or r_π here because they are for signals (AC) only.

Alternative calculation : The +10 V power supply supplies a total current of 1.29 mA through R_c plus 0.5 mA $\left(\frac{+10V}{10k+10k}\right)$ through the base resistors. Total : 1.79 mA. $P = V \cdot I =$ 17.9 mW

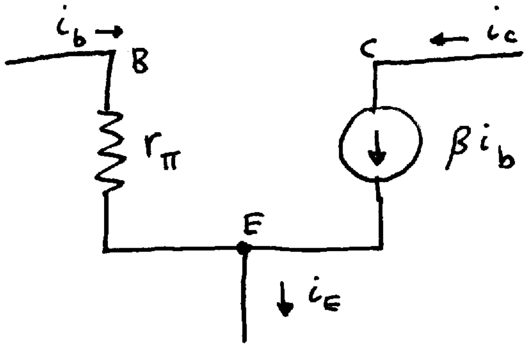
N: Small-signal models of transistors and amplifiers.

Although electronic circuits can be analyzed without the help of small-signal models, in some cases those small-signal model can help to understand things. Examples of npn BJT models

"base oriented" = HYBRID-π

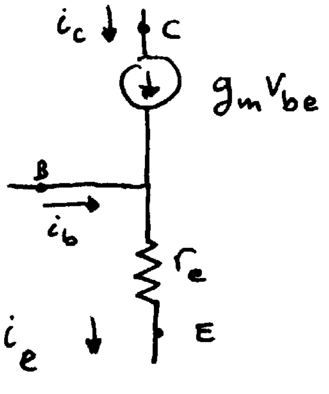


Hybrid-π model with voltage-controlled current source

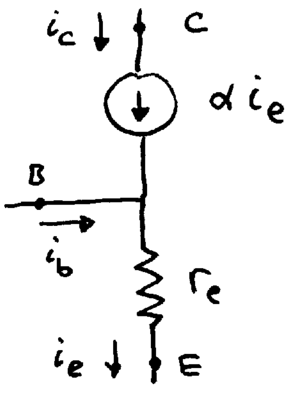


Hybrid-π model with current controlled current source

"emitter oriented" = T MODEL

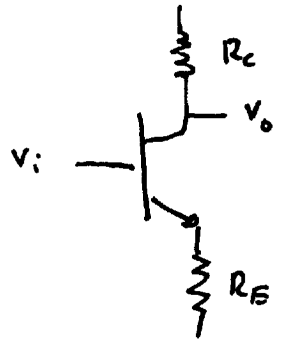


T model with voltage-controlled current source



T model with current controlled current source

0: BASIC BJT AMPLIFIERS

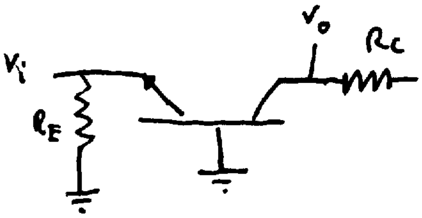


COMMON - EMITTER AMPLIFIER (CEA)

signal gain : $\frac{v_o}{v_i} = -\frac{R_c}{r_e + R_E}$

$$r_{in} = (\beta+1)(r_e + R_E)$$

$$r_{out} = R_c$$

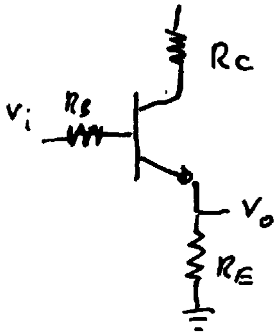


COMMON - BASE AMPLIFIER (CBA)

gain $\frac{v_o}{v_i} = +\frac{R_c}{r_e}$

$$r_{in} = r_e // R_E$$

$$r_{out} = R_c$$



COMMON - COLLECTOR AMPLIFIER (CCA)

gain $\frac{v_o}{v_i} = R_E/(r_e + R_E) \approx 1$

$$r_{in} = (\beta+1)(r_e + R_E) + R_B$$

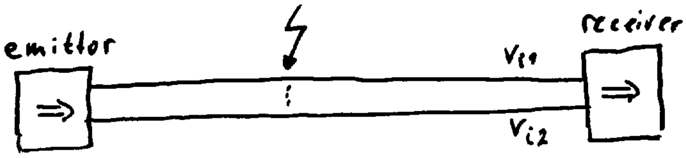
$$r_{out} = R_E // \left(\frac{R_B}{\beta+1} + r_e\right)$$

CHAPTER 1 : DIFFERENTIAL PAIR

A differential amplifier has two inputs and two outputs. The difference voltage at the output is proportional to the difference voltage at the input

$$V_{o2}-V_{o1} = (V_{i2}-V_{i1}).A$$

The advantage is obvious. For transmission lines, the noise is reduced



Whatever noise is introduced in the line, this noise is normally equal in both line. (The noise is "correlated"). Therefore, the difference is zero. $V_{i2}-V_{i1}=0$. All noise will be rejected. Most communication lines work like this (twisted-pair network cables, etc.). The differential amplifier is therefore one of the most important electrical circuits.

An ideal diferencial amplifier thus reject all signals that appear on both terminals. We can define this in the common-mode rejection ratio (CMRR). This is the ratio of the gain at common mode

$$A_{cm} = \frac{V_o}{V_i} \qquad \text{with } V_o = V_{o1} = V_{o2} \text{ and } V_i = V_{i1} = V_{i2}$$

und the differencial gain

$$A_{dm} = \frac{V_{o2}-V_{o1}}{V_{i2}-V_{i1}}$$

Then

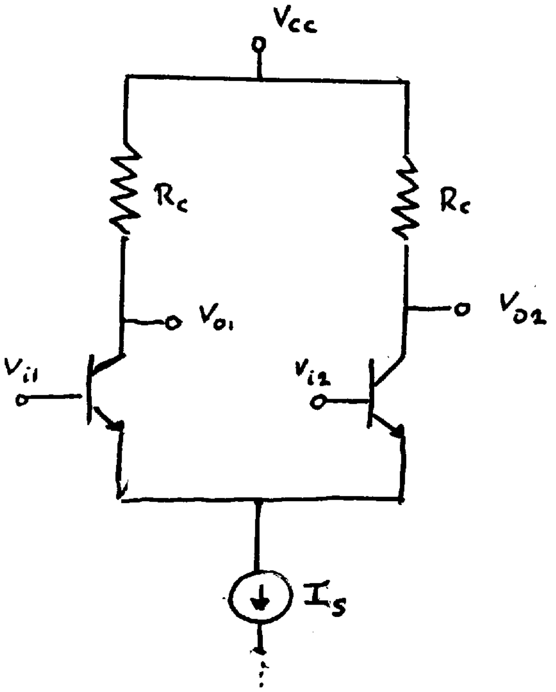
$$\text{CMRR} = \left| \frac{A_{dm}}{A_{cm}} \right|$$

This should be as high as possible. The CMRR is an important parameter to qualify a differencial amplifier. Other aspects are cost (number of components), power consumption and possibility to fabricate in integrated circuits.

One of the most popular is the differential pair (par diferencial).

Par Diferencial

(Sedra : ch 6
Bogart : ch 12
Moura : ch 1)



A basic differential pair consists of two (npn) transistors, a current source (I_s) and current-to-voltage converters R_c

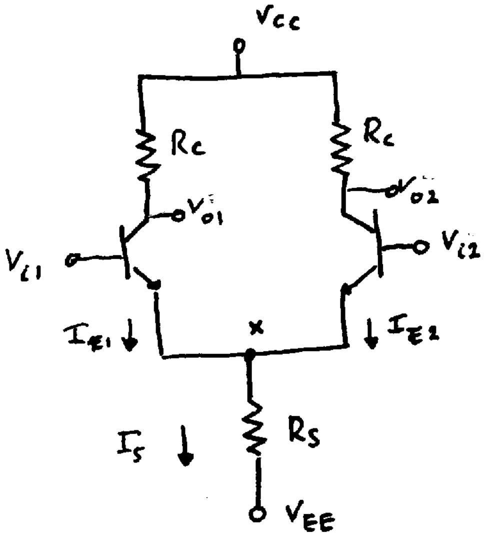
We will calculate the gains (A_{cm} and A_{dm}) and CMRR for different versions of this basic model

1 : I_s defined by a simple resistance

2 : I_s a current source made with a transistor

3 : current-to-voltage converters made of pnp transistors

Model 1



⟸ simple DP current source is a resistance. Why can we call it a current source? Because the DC voltage at x (the emittors of both transistors) is ~ −0.7 V (assuming small signals at V_{i1} and V_{i2} and no DC components)

Thus $I_S = (-0.7 - V_{EE}) / R_S$

$R_C = 3.3\,K\Omega$, $R_S = 10\,k\Omega$
$V_{CC} = +10\,V$, $V_{EE} = -10\,V$

<u>Polarization</u> : $V_{i1} = V_{i2} = 0\,V$

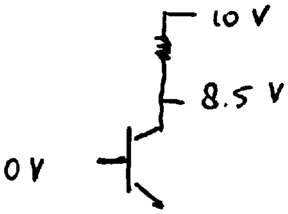
$V_x = -0.7\,V$. $I_S = (-0.7 + 10) / 10\,k\Omega = 0.93\,mA$.

Because of symmetry : $I_{E1} = I_{E2} = \frac{1}{2} I_S = 0.465\,mA$.

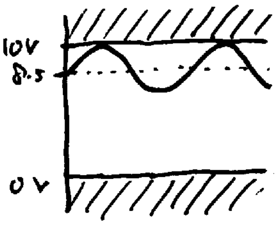
$\beta = 100 \Rightarrow \alpha \approx 1$. $I_{C1} = I_{C2} = 0.465\,mA \Rightarrow V_{01} = V_{02} = 10\,V - 0.465\,mA \cdot 3.3k$
$= 8.47\,V$

$r_e = V_T / I_{E1} = 56\,\Omega$

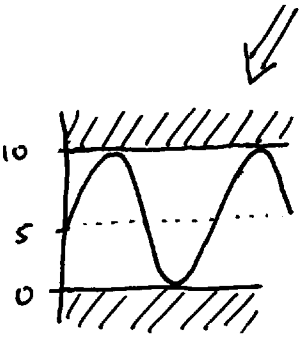
Did we design this DP well? No!



The output can swing up to 10 V and down to ~ 0 V. This gives us a maximum



output signal of ~ 1.5 V amplitude. Had we designed the DP to have a bias at 5 V, the maximum output signal would have been 5 V.

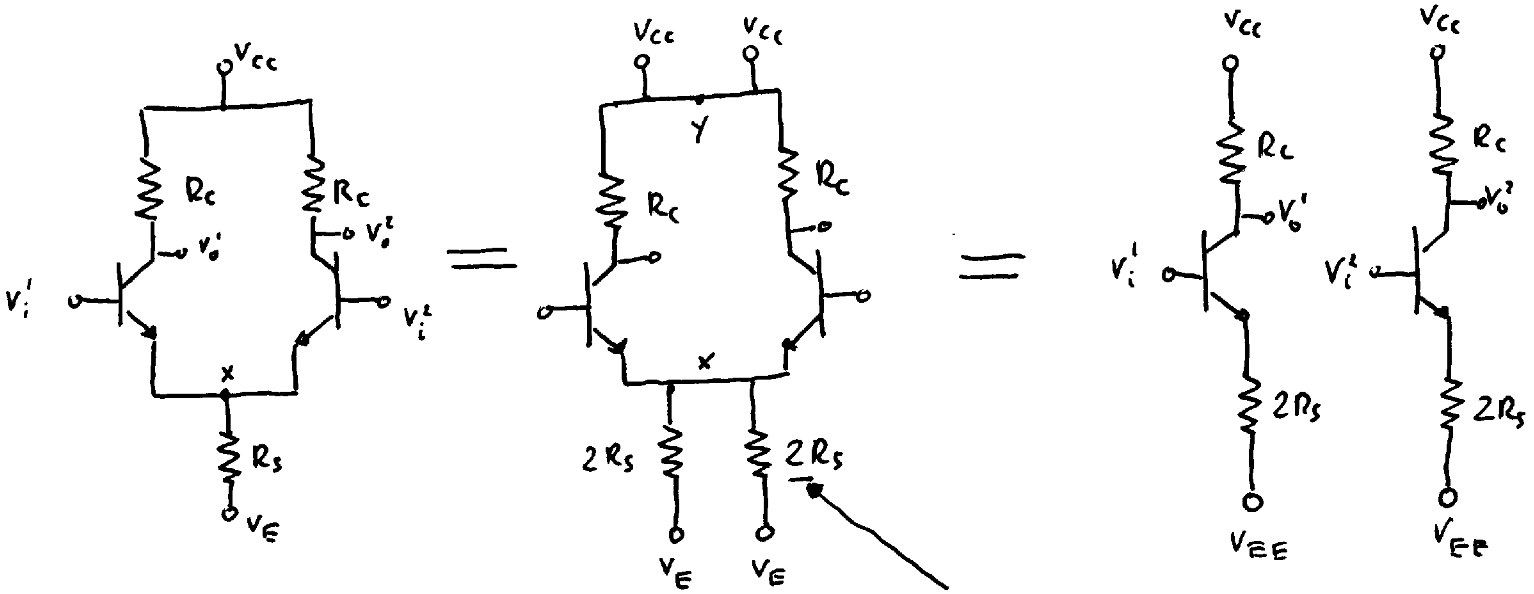


Options : increase R_C (to 10.8 kΩ) or decrease R_S (to 3.1 kΩ)

But... let's continue with this poor DP

Common Mode Gain (A_{cm}).

since at all points on the left the voltages and currents are exactly the same as their counterpoints on the right, we can use a trick of symmetry :



At x,y no current flows (because of symmetry) → we can cut this

note factor 2

To calculate $A_{cm} = \frac{V_o^1}{V_i^1}$ is no problem. This is a normal common emittor amplifier :

$$\boxed{A_{cm} = -\frac{R_c}{r_e + 2R_s}} = -\frac{3.3\,k\Omega}{56\,\Omega + 2\times 10\,k\Omega} = -0.165$$

Alternative way of analyzing (for ~~those~~ who don't like to use tricks and laws of physics) similar to p.5 of ch.0

$$A_{cm} = \left.\frac{\partial V_{o1}}{\partial V_{i1}}\right|_{V_{i1}=V_{i2}}$$

✓ $I_s = I_{E1} + I_{E2} = 2\,I_{E1}$

$V_x = V_{E1} = V_{E2}$

$$\frac{\partial I_{B1}}{\partial V_{B1}} = \frac{1}{\beta+1}\frac{\partial I_{E1}}{\partial(V_{E1}+0.7V)} = \frac{1}{\beta+1}\frac{\frac{1}{2}\partial I_s}{\partial V_x} = \frac{1}{2(\beta+1)}\cdot\frac{1}{R_s}$$

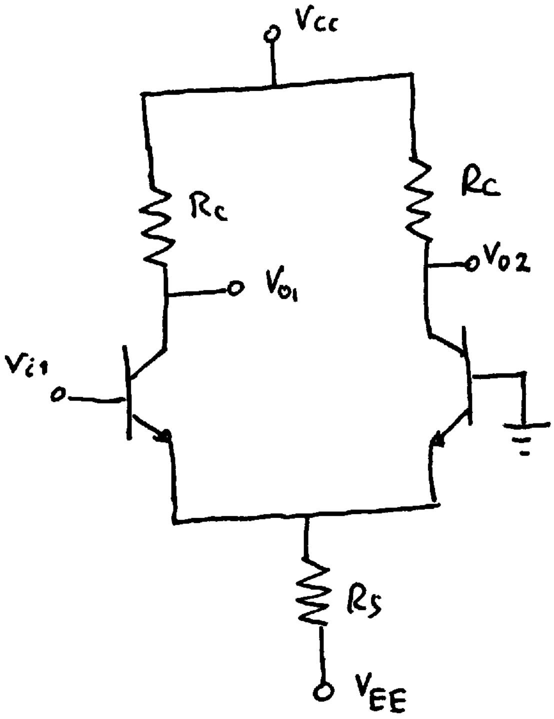
↑ V_{BE} constant ⇒ $r_e = 0$

V_{BE} not constant ⟶ $\frac{1}{(\beta+1)}\;\frac{1}{r_e + 2R_s}$

$$\partial V_o^1 \qquad \frac{\partial V_{o1}}{}\cdot\frac{\partial I_{C1}}{}\cdot\frac{\partial I_{B1}}{} = -R_c\cdot\beta\cdot\frac{1}{(\beta+1)}\frac{1}{r_e+2R_s} = -\frac{R_c}{r_e+2R_s}$$ q.e.d.

Differential Mode Gain (A_{dm})

$A_{dm} = \frac{V_{o2} - V_{o1}}{V_{i2} - V_{i1}}$. For our analysis we will connect the second input to ground ($v_{i2} = 0$)

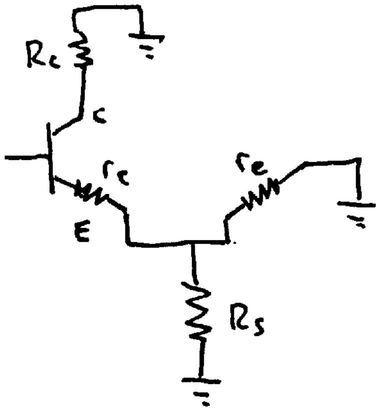


⇐ DP in DM

$$A_{dm} = \frac{V_{o1}}{V_{i1}} - \frac{V_{o2}}{V_{i1}}$$

↗ single-ended common-mode gain

To calculate the gain $\frac{V_{o1}}{V_i}$ is easy: All resistance at collector divided by all resistance at emittor.

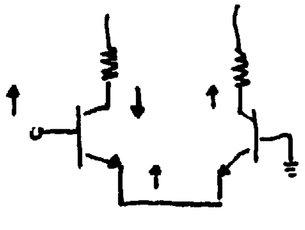


$$\frac{V_{o1}}{V_{i1}} = - \frac{R_c}{r_e + r_e // R_S} \sim - \frac{R_c}{2 r_e}$$

This is the single-ended DM gain A_{dm}^{se}

The other output has the same gain, but with positive sign

$$\frac{V_{o2}}{V_{i1}} = + \frac{R_c}{2 r_e} \quad \text{why ?}$$



V_{i1} increases → V_{BE1} increases → I_{c1} increases → V_{o1} decreases → negative sign

V_{i1} increases → V_{E1} increases (albeit a little less) → V_{BE2} decreases → I_{B2} decreases → I_{C2} decreases → V_{o2} increases.

Another way to understand this :

Because we have a <u>current source</u>, the current is constant $I_{E1} + I_{E2} = \text{constant} \Rightarrow i_{e1} = -i_{e2} \Rightarrow$

$$\left.\begin{aligned} i_{c2} &= -i_{c1} \\ v_{o2} &= -R\,i_{c2}\,,\quad v_{o1} = -R\,i_{c1} \end{aligned}\right\}\ v_{o2} = -v_{o1}$$

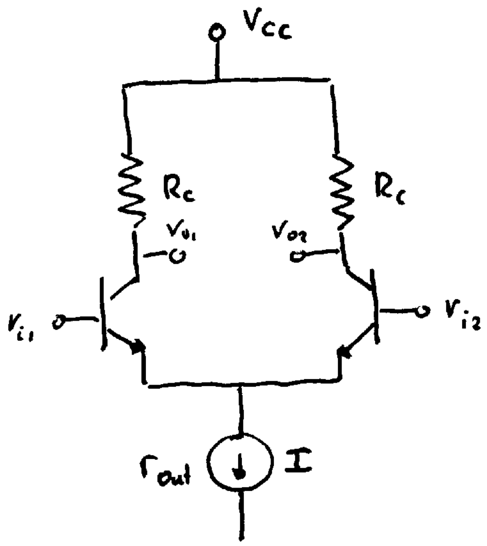
$$\boxed{A_{dm}} = \frac{v_{o1}}{v_{i1}} - \frac{v_{o2}}{v_{i1}} = -\frac{R_c}{2r_e} - \frac{R_c}{2r_e} = \boxed{-\frac{R_c}{r_e}} = -58.9$$

and

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| = \left|\frac{-R_c/r_e}{-R_c/(r_e+2R_s)}\right| = \frac{2R_s+r_e}{r_e} = 358$$

To improve the CMRR we should increase R_s in some way. The best way to do it is to replace R_s with a current source. Theoretically $r_{out} = \infty$ (ideal current source)

Model 2



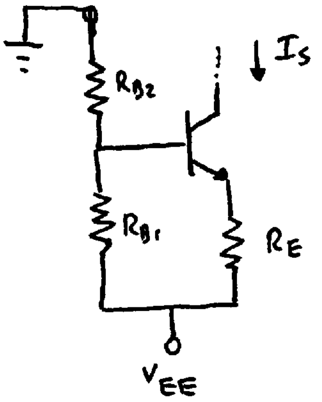
$$A_{cm} = -\frac{R_c}{r_e + 2r_{out}} = 0 \quad \text{(ideal)}$$

$$A_{dm} = -\frac{R_c}{r_e}$$

$$CMRR = \frac{2r_{out}+r_e}{r_e} = \infty \quad \text{(ideal)}$$

The problem now boils down to : how to make a current source with maximum r_{out}. The "current source" of model 1 is rather poor and has an r_{out} of R_s ($\sim k\Omega$'s)

Current source:



r_i of transistor $\gg R_{B1}, R_{B2} \Rightarrow$

$$V_B = \frac{R_{B2}}{R_{B1}+R_{B2}} \cdot V_{EE}$$

$$V_E = V_B - 0.7$$

$$I_E = (V_E - V_{EE})/R_E = \left[\frac{R_{B2}}{R_{B1}+R_{B2}} \cdot V_{EE} - 0.7 - V_{EE}\right] /$$

"Easy calculation" will show that the output resistance of this current source is

$$r_{out} = r_o + \left[(r_\pi + R_B) // R_E \right] + \frac{r_o \beta R_E}{r_\pi + R_B + R_E}$$

with r_o equal to output resistance of transistor $\left(\frac{V_A}{I_E}\right)$

$r_\pi = (\beta+1) r_e$

$R_B = R_{B1} // R_{B2}$

Example : $R_{B1} = 10\ k\Omega$, $R_{B2} = 10\ k\Omega$, $R_E = 4.7\ k\Omega$

$r_o = 200\ k\Omega$, $\beta = 100$, $r_\pi = 2\ k\Omega$, $V_{EE} = -10\ V$

$V_B = -5\ V$, $V_E = -5.7\ V$, $I_E = 0.91\ mA \Rightarrow \boxed{I_s = 0.91\ mA}$

$$r_{out} = 200\ k\Omega + \left[(2 k\Omega + 5 k\Omega) // 4.7\ k\Omega \right] + \frac{200\ k\Omega \times 100 \times 4.7\ k\Omega}{2 k\Omega + 5 k\Omega + 4.7\ k\Omega}$$

$$= 200\ k\Omega + 2.8\ k\Omega + 100 \times 80\ k\Omega = \boxed{8.2\ M\Omega}$$

OUR DP WITH THIS SOURCE:

$A_{cm} = -2.0\ 10^{-4}$

$A_{dm} = -58.9$ (unchanged)

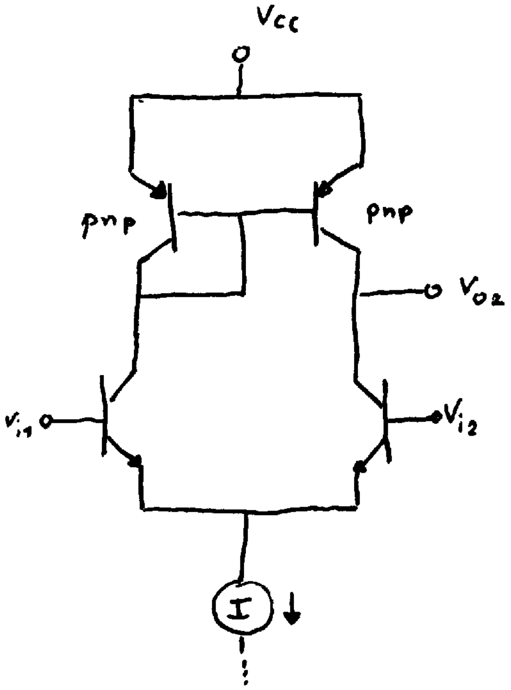
$CMRR = \left| \frac{-58.9}{-2.0\ 10^{-4}} \right| = 2.9\ 10^5$ (800 times better!)

To further increase the CMRR we can try to increase the differential gain A_{dm}. Remember

$$A_{dm} = -R_c/r_e$$

So we should increase R_c. This can be done by replacing the collector resistances by (pnp) transistors, whose output resistance is r_o (100's kΩ):

Differential pair with active load



r_o of pnp trans. 200 kΩ

r_{out} of current source 8.2 MΩ

$$A_{cm} = -\frac{r_o}{r_e + 2\,r_{out}}$$

$$A_{dm} = -\frac{r_o}{r_e}$$

$$CMRR = \frac{2\,r_{out} + r_e}{r_e} \quad \text{(unchanged!)}$$

In our example $r_e = 56\ \Omega$:

$$A_{cm} = -\frac{200\ k\Omega}{56\ \Omega + 2 \times 8.2\ M\Omega} = -1.22\ 10^{-2}$$

$$A_{dm} = -\frac{200\ k\Omega}{56\ \Omega} = 3570$$

$$CMRR = 2.9\ 10^{5} \quad \text{unchanged!}$$

The advantage of the active load is not a higher CMRR. So why are all DP's implemented with active load?

The reason is the fact that there are no resistances in the circuit (as we will see, the current source can be made by current mirrors). In integrated circuits it is very difficult to make resistances (where it is easy to make transistors).

One final observation. We used here the single-ended gain / output. Only v_{o2}. In fact, the gain at v_{o1} is much lower. When we connect v_{i2} to ground (differential mode):

$$\left.\begin{array}{l} \dfrac{v_{o1}}{v_{i1}} = -\dfrac{r_\pi^p // r_\pi^p}{2r_e^n} \approx -\dfrac{\beta}{4} \\[3ex] \dfrac{v_{o2}}{v_{i1}} = \dfrac{r_o^p}{2r_e^n} \end{array}\right\} \quad A_{dm} \approx -\frac{r_o^p}{2r_e}$$

(r_π^p is of pnp, r_e^n of npn tr.)
(r_o^p is of pnp)

common mode ($v_{i1} = v_{i2}$)

$$\left.\begin{array}{l} \dfrac{v_{o1}}{v_{i1}} = -\dfrac{r_\pi^p // r_\pi^p}{r_e^n + 2r_o^s} \\[3ex] \dfrac{v_{o2}}{v_{i1}} = -\dfrac{r_o^p}{r_e^n + 2r_o^s} \end{array}\right\} \quad A_{cm} \approx \frac{r_o^p}{2r_o^s}$$

(r_o^s is of current source)

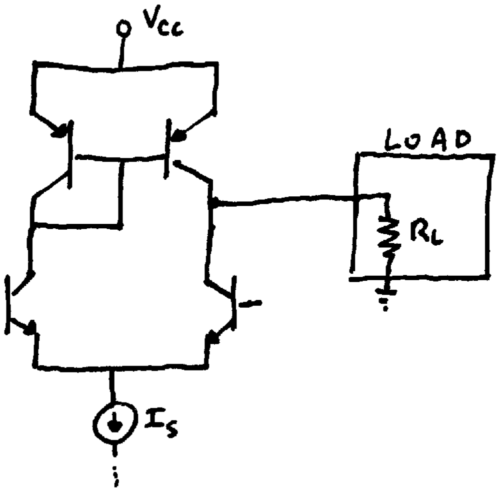
$$CMMR = \frac{r_o^s}{r_e}.$$

(A factor 2 less than of p. 8)

A fact that is not found back in textbooks.

One more observation about the DP with active load:

This is a strange amplifier, because the input signal is <u>voltage</u>, but the output signal is rather <u>current</u>. (ideally, $r_o^p = \infty$). This amplifier only works when connected to something. In that case the gain becomes limited by the load resistance.



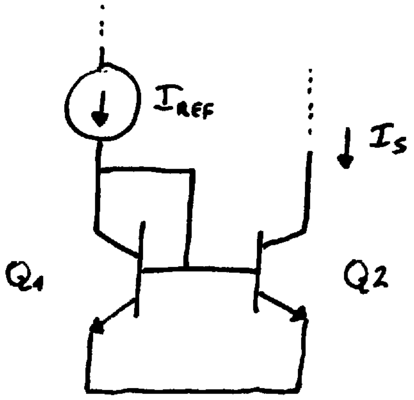
$$A_{cm} = -\frac{R_L}{r_e + 2 r_{out}}$$

$$A_{dm} = -\frac{R_L}{r_e}$$

$$CMRR = \frac{2 r_{out} + r_e}{r_e} \quad \text{(unchanged)}$$

CURRENT SOURCES

We already used a current source (on p.7) and, as a matter of fact, the active load in the previous amplifier is also a current source, a so-called current mirror. It is called a mirror, because the current in one leg (the right in this case) is equal to the current in the other leg (left in this case). The reason why this is so is easy to see. Consider the next circuit of a current mirror:



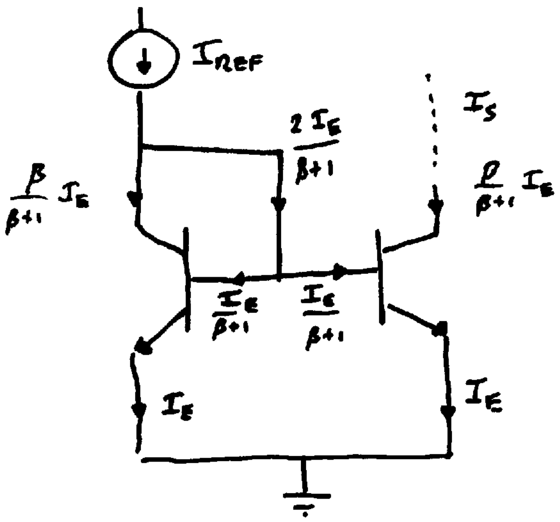
I_s is equal to I_{REF} ! why

Imagine I_{REF} is from ideal current source. This current is 'pushed' through transistor Q1. ("at all cost"). Therefore, a voltage drop V_{BE1} is induced at the base-emittor junction. According to Ebers-Moll (p.2 ch.0) for a diode $I_{E1} = I_{REF} = I_0 \exp[V_{BE}/V_T]$

$$\Rightarrow \quad V_{BE1} = V_T \ln(I_{REF}/I_0)$$

The voltage drop at the right transistor is equal (same base and same emittor voltages). $V_{BE2} = V_{BE1}$. Therefore, $I_{C2} = I_{C1} = I_{REF}$. (using Ebers-Moll).

Here we have assumed an infinite β. For finite β we can see that a little bit of the current I_{REF} is lost for biasing the transistors.



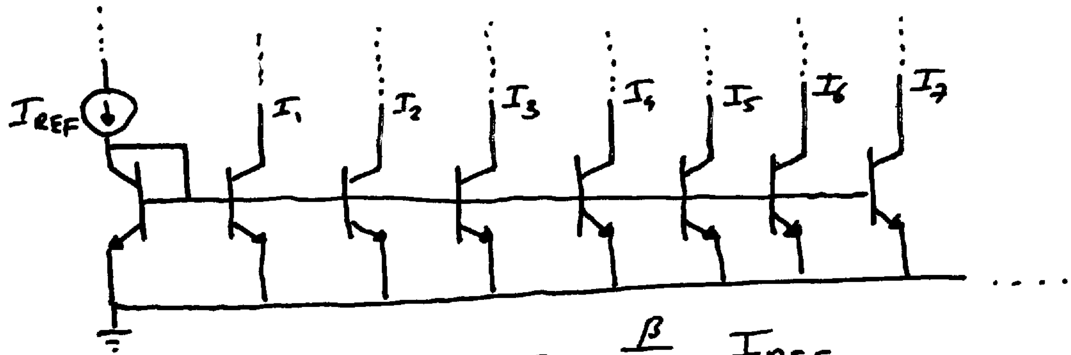
$\leftarrow$ $I_s = \frac{\beta}{\beta+1} I_E$

$$I_{REF} = \frac{\beta}{\beta+1} I_E + \frac{2}{\beta+1} I_E$$

$$\Rightarrow I_s = I_{REF} \left(\frac{\beta}{\beta+2} \right)$$

We started with a current source (I_{REF}) and ended with

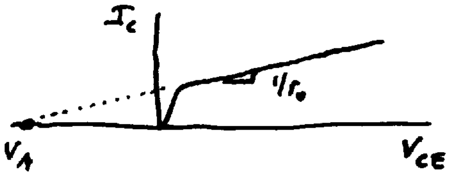
Multi - current - source



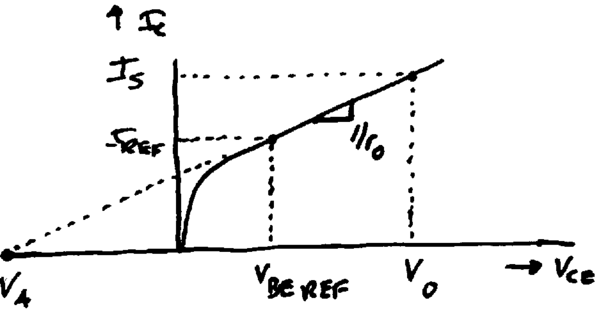
$$I_1 = I_2 = \dots = \frac{\beta}{\beta+1+n} I_{REF}$$

The output resistance of a current mirror is r_o ($\sim V_A/I_c$)

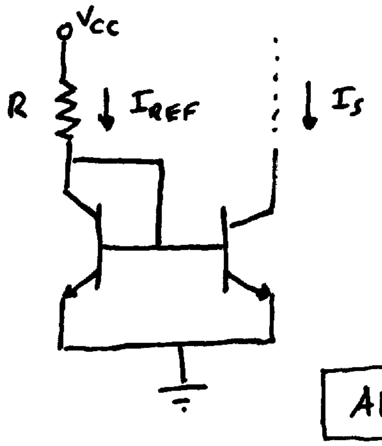
This causes another non-linearity. Since the current I_{cs} depends on the collector voltage, the source current depends on the output voltage. and is not necessarily equal to I_{REF} ($V_{CREF} = V_{BREF} \neq V_{C1}$).



$$I_S = I_{REF} + (V_o - V_{BE,REF})/r_o = I_{REF}\left(1 + \frac{V_o - V_{BE}}{V_A}\right)$$



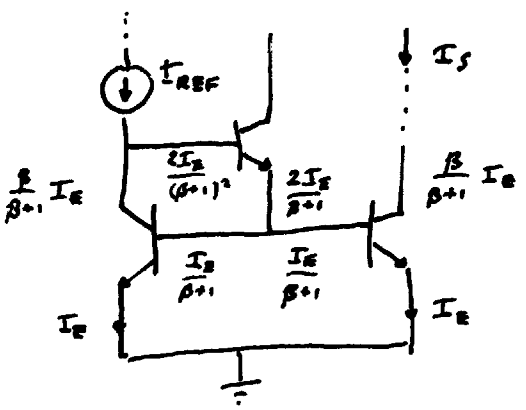
Other current sources / mirrors :



- $I_S = \frac{V_{cc} - 0.7}{R}$
- $r_{out} = r_o$

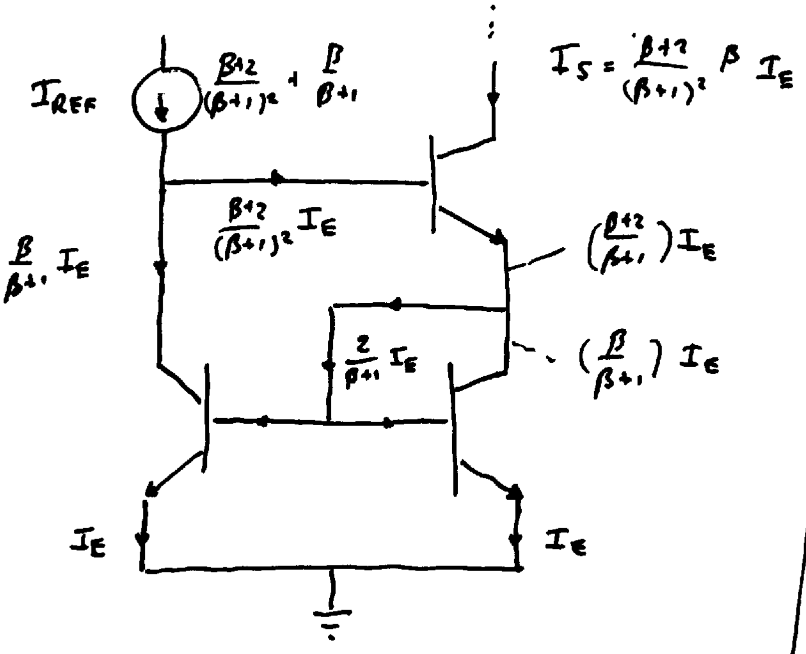
(suffers from "β" and "V_o" effects)

ADDING R TURNS A MIRROR INTO A SOURCE!



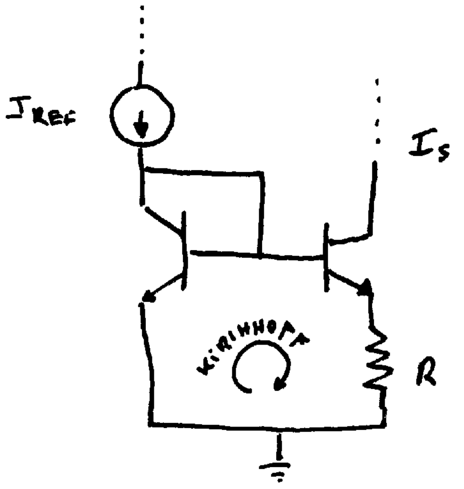
- $I_S \approx I_{REF}$
- $r_{out} = r_o$
- doesn't suffer (so much) from "β" effect (only $2I_E/(\beta+1)^2$ is lost)

WILSON CURRENT MIRROR



- $I_S = I_{REF} \left(\frac{1}{1 + 1/(\beta^2 + 2\beta)} \right)$
- reduced "β" effect
- $r_{out} \approx \frac{\beta r_o}{2}$

WIDLAR CURRENT SOURCE



$$V_{BEREF} = V_T \ln\left(\frac{I_{REF}}{I_o}\right)$$

$$V_{BES} = V_T \ln\left(\frac{I_s}{I_o}\right)$$

Kirchoff : $V_{BEREF} - V_{BES} - I_s R = 0$

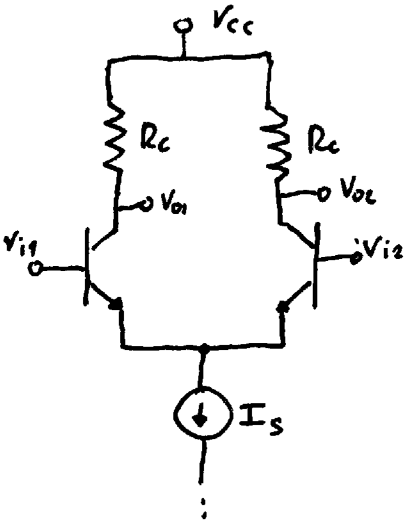
$$I_s R = V_T \ln\left(\frac{I_{REF}}{I_s}\right)$$

- R very small (still possible with integr. circuits)

  $I_{REF} \approx I_s$

LARGE - SIGNAL ANALYSIS OF D.P.

until now it was assumed that we worked in the linear region of the D.P. That is $V_o \sim V_i$. Up to what point was this justified?



using Ebers - Moll

$V_{BE1} = V_{i1} - V_E$

$V_{BE2} = V_{i2} - V_E$

$$I_{E1} = I_o \exp\left(V_{BE1}/V_T\right)$$

$$I_{E2} = I_o \exp\left(V_{BE2}/V_T\right)$$

$$\Longrightarrow \frac{I_{E1}}{I_{E2}} = \exp\left(\frac{V_{i1}-V_{i2}}{V_T}\right)$$

also : $I_{E1} + I_{E2} = I_s$

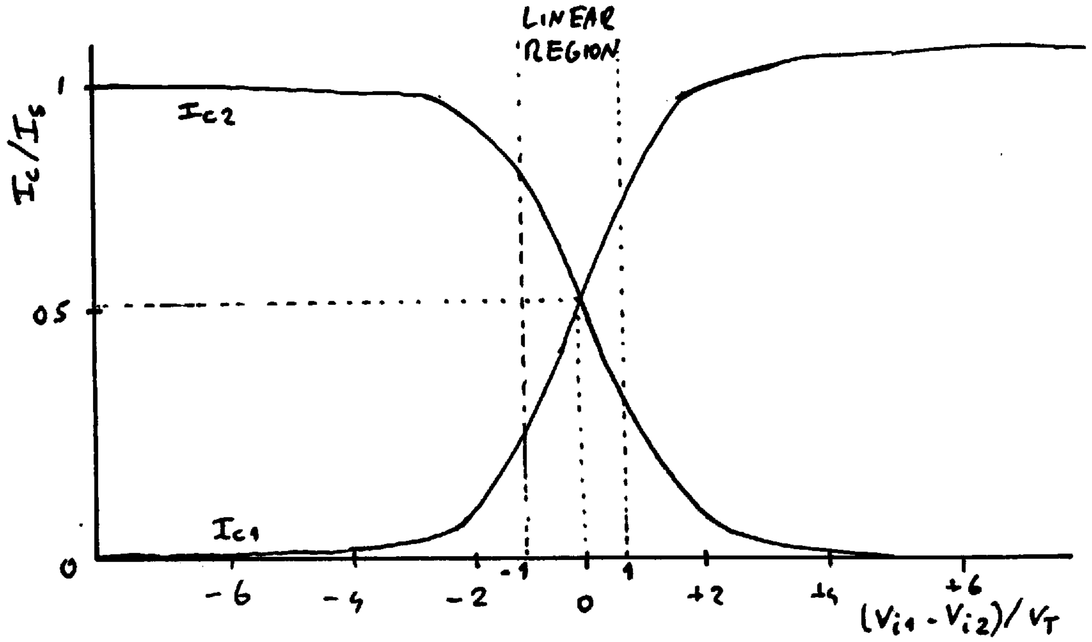
$\Longrightarrow$

$$I_{E1} = \frac{I_s}{1 + \exp\left(\frac{V_{i2}-V_{i1}}{V_T}\right)} , \quad I_{E2} = \frac{I_s}{1 + \exp\left(\frac{V_{i1}-V_{i2}}{V_T}\right)}$$

Assuming $\alpha \simeq 1$ $\quad I_{C1} = I_{E1}$, $I_{C2} = I_{E2}$

The resistances R_c are current-to-voltage translators

$$V_{o1} = V_{cc} - \frac{R_c I_s}{1 + \exp\left(\frac{V_{i2} - V_{i1}}{V_T}\right)} \quad , \quad V_{o2} = V_{cc} - \frac{R_c I_s}{1 + \exp\left(\frac{V_{i1} - V_{i2}}{V_T}\right)}$$



The amplifier is linear oproximately for differential voltages not exceeding V_T !

$$\boxed{\text{USE INPUT SIGNALS OF} \sim 20\,\text{mV}}$$

CHAPTER 2

FREQUENCY RESPONSE

ch. 7 Sedra

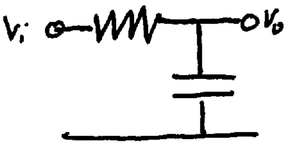
One of the most important parameters of electronic circuits is the frequency response. Normally we want as high a bandwidth as possible, but not always. Sometimes we want to cut off the DC part (for decoupling or to reduce power consumption or noise). Sometimes we want to limit the high-frequencies (to avoid oscillations). sometimes both (bandpass amplifier).

For advanced circuits (with many capacitors and inductors) the calculation is very complicated. The best thing is then to use electronics simulators such as Electronics Workbench an P-Spice.

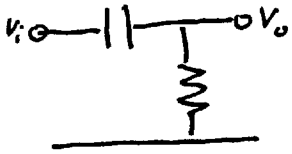
However, it is very usefull to get an idea of the frequency response. In order to do that we will simplify our analysis. We will look for individual cut-off frequencies and then combine these in the final frequency response. Thus, the strategy becomes very simple:

> Find filters like the LP and HP filters presented in Chapter 0

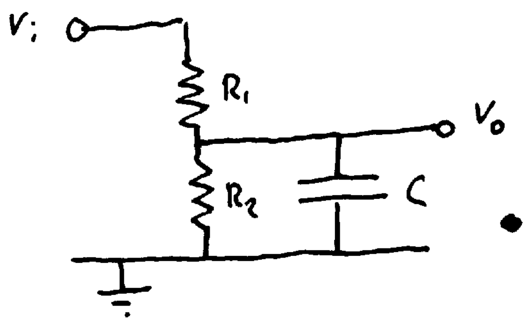
As an example:



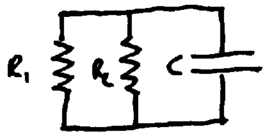
LPF



HPF



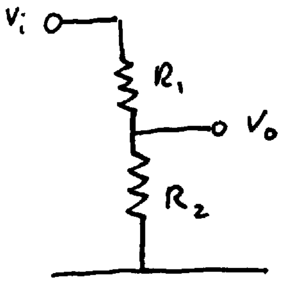
- This filter looks like a low-pass filter
- Because it has only 1 condensator, it has only one cut-off frequency
- To find this frequency we have to find the effective resistance this C sees. For this, we connect the input to ground and we get this



$\swarrow$ circuit. $R_{eff} = R_1 // R_2 \Rightarrow \omega_0 = C\,(R_1 // R_2)$

$$\Rightarrow \quad T(s) = \frac{A_{DC}}{1 + s / \frac{1}{(R_1 // R_2) C}}$$

- The DC gain A_{DC} can be found by considering the condensator as open-circuit :



$$A_{DC} = \frac{R_2}{R_1 + R_2}$$

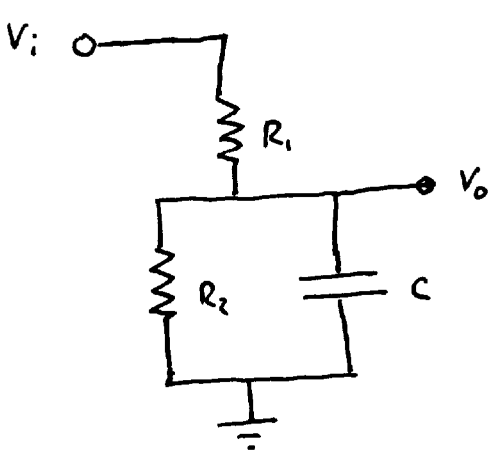
Thus

$$T(s) = \frac{R_2 / (R_1 + R_2)}{1 + s / \frac{1}{(R_1 // R_2) C}}$$

with $R_1 // R_2 = R_1 R_2 // (R_1 + R_2)$ this becomes

$$T(s) = \frac{1 / R_1 C}{s + 1 / (R_1 // R_2) C} \qquad (*)$$

The method we learned at Circuit Analysis results (of course) in the same :



This is a voltage divider with the impedance of a condensator as $\frac{1}{sC}$:

$$\frac{V_o}{V_i} = \frac{(\frac{1}{sC} // R_2)}{(\frac{1}{sC} // R_2) + R_1}$$

After some rearranging of terms the same (*) expression emerges. Try it.

Which method do you prefer ?

Bode plots

As a simplification of our analysis we will look for simple filters in our circuit: Either LPF's or HPF's. In other terms, we will look for single-pole (LPF) or single-zero (HPF) filters, so-called first-orde functions. On basis of this we will draw a Bode plot observing the following rules:

- we start with a horizontal line ($A = A_0$)
- Every pole (LPF) introduces a cut-off frequency <u>above</u> which the slope is changed by -20 dB/dec.
- Every zero (HPF) introduces a cut off frequency <u>below</u> which the slope is changed by +20 dB/dec

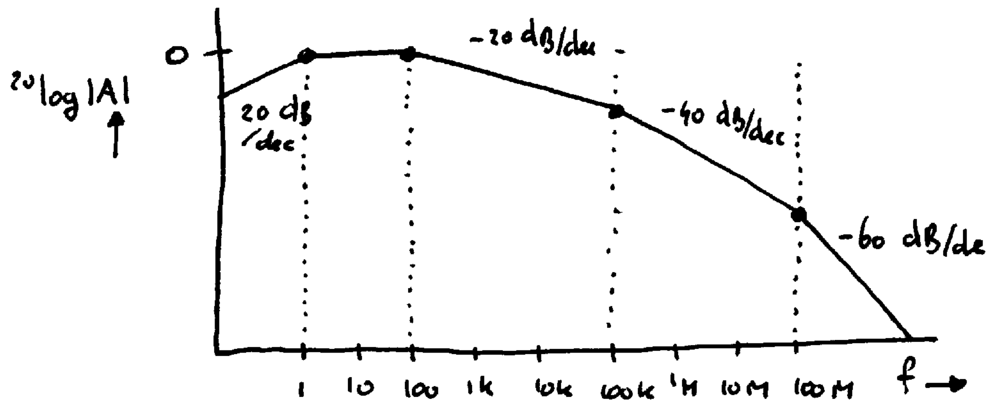
At the phase plot:

- we start with 0°
- The phase changes -90° in approximately 2 decades (factor 100 in frequency) around the cut-off frequency introduced by every pole (LPF). -45° before f_c, -45° after f_c. At f_c : $\Delta\varphi = -45°$
- The phase changes +90° in approximately 2 decades around the cut-off frequency introduced by every zero (HPF). +45° before f_c, +45° after f_c. At f_c : $\Delta\phi = +45°$

Example :

what is the Bode plot of a filter (passive, $A_{max} = 1$) with three poles (100 Hz, 100 kHz, 100 MHz) and one zero (1 Hz) ?

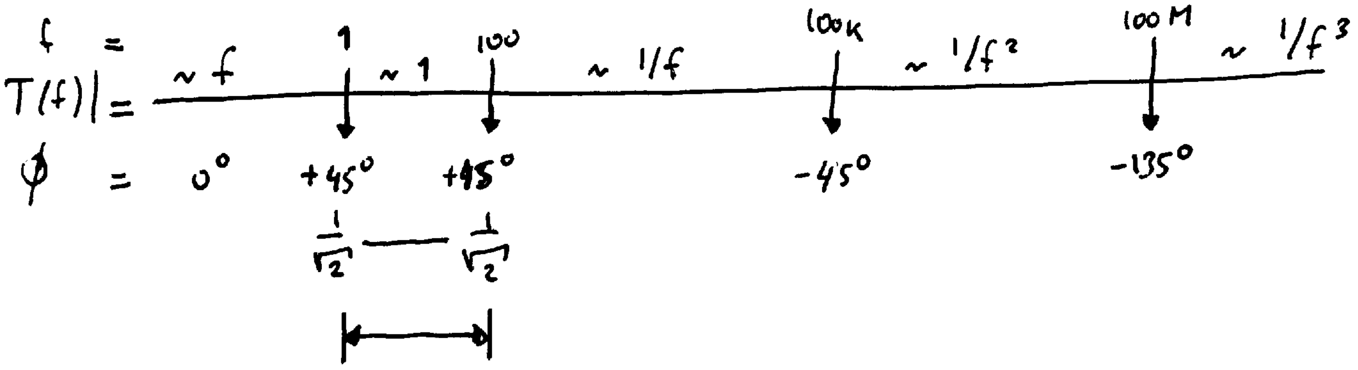
The mid band gain is 1 (with 90° shift).



Note that these are approximations

The real functions are smoother

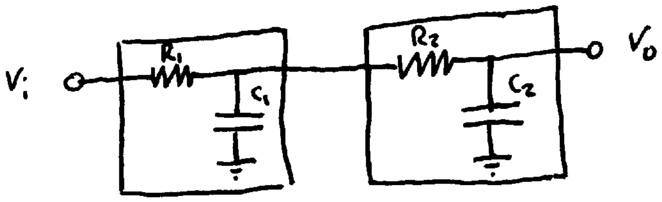
$$T(f) = \frac{1}{(1 + jf/100)} \cdot \frac{1}{(1 + jf/10^5)} \cdot \frac{1}{(1 + jf/10^8)} \cdot \frac{1}{(1 + 1/jf)}$$



Bandwidth of filter : 100 - 1 = 99 Hz

This is a band - pass filter.

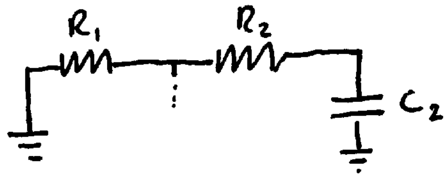
Warning 1 : Don't be misled



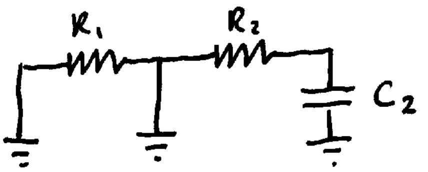
What are the two poles of this filter ?

simple thought : $\omega_1 = \frac{1}{R_1 C_1}$, $\omega_2 = \frac{1}{R_2 C_2}$. wrong !

For instance : we have to find the effective R that C_2 sees, while connecting V_i to ground. This might be $R_1 + R_2$ (if C_1 is considered open-circuit) or just R_2 (if C_1 is considered short-circuit). Which one is correct depends on values of R_1 R_2 C_1 and C_2 .



or

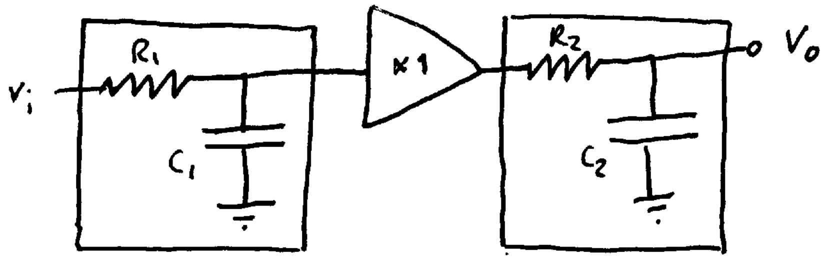


If the pole of filter 1 is much lower than that of the pole we are trying to find for filter 2 then we use the right circuit, because $f_2 >> f_1$ $\Rightarrow$ The first filter is already blocking → C_1 is effectively a short circuit. If the pole of filter 1 is much higher, then we use the left circuit and find $\omega_2 = \frac{1}{(R_1 + R_2) C_2}$

There are many ways to go to Rome. Many ways to analyze. In case of doubt → Spice

Warning 2 : multi pole/zero

Even if the individual filters are nicely decoupled, as in the figure below, we have to be carefull.



note : ideal amplifier (op amp) has $r_{in} = \infty$, $r_{out} = 0$

Imagine $R_1 = R_2 = R$ and $C_1 = C_2 = C$. Two identical filters. what is the cut-off frequency of the total circuit?

$$\omega_1 = \frac{1}{RC} \quad , \quad \omega_2 = \frac{1}{RC} = \omega_1 \Rightarrow \omega_{tot} = \frac{1}{RC} \ ? \quad \text{wrong!}$$

Remember : at the cut off frequency the amplitude is $\frac{1}{\sqrt{2}}$ of mid band amplitude. Per definition!

At $\omega = \omega_1$: $|T(\omega_1)| = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$, because

$$|T(\omega)| = \frac{1}{\sqrt{1 + \omega^2/\omega_1^2}} \cdot \frac{1}{\sqrt{1 + \omega^2/\omega_2^2}} \Rightarrow \frac{1}{2}$$

Easy calculation shows $(\omega_2 = \omega_1)$:

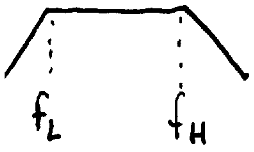
$$|T(\omega_{tot})| = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{1 + \omega_{tot}^2/\omega_1^2} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\omega_{tot} = \sqrt{\sqrt{2} - 1}\ \omega_1 \sim 0.64\ \omega_1$$

So, if, for instance, we link two 100 Hz LPF's together, the total cut-off frequency is 64 Hz.

In the same way: for two equal VHPF's $\omega_{tot} = 1.55\ \omega_1$

Normally what is interesting is to find the pass-band. That is to say the lower and upper frequencies of band that has a gain larger than $\frac{1}{\sqrt{2}}$ times the maximum gain. $f_L \dots f_H$.



* When the individual lower cut-off frequencies $f_{L1}, f_{L2} .. f_{LN}$ are well seperated, or when at least the higher one is well seperated, then

$$f_L \approx MAX(f_{Li})$$

* The same accounts for the higher cut-off frequency f_H (when "well seperated")

$$f_H \approx MIN(f_{Hi})$$

("well seperated" means more-or less a factor 4)

For instance:

$f_{L1} = 1$ Hz, $f_{L2} = 2$ Hz, $f_{L3} = 2.4$ Hz, $f_{L4} = 30$ Hz

$f_{H1} = 200$ kHz, $f_{H2} = 3$ MHz, $f_{H3} = 3.3$ MHz

$\Rightarrow f_L \sim 30$ Hz, $f_H \sim 200$ kHz

* when not well seperated it can easily be calculated as

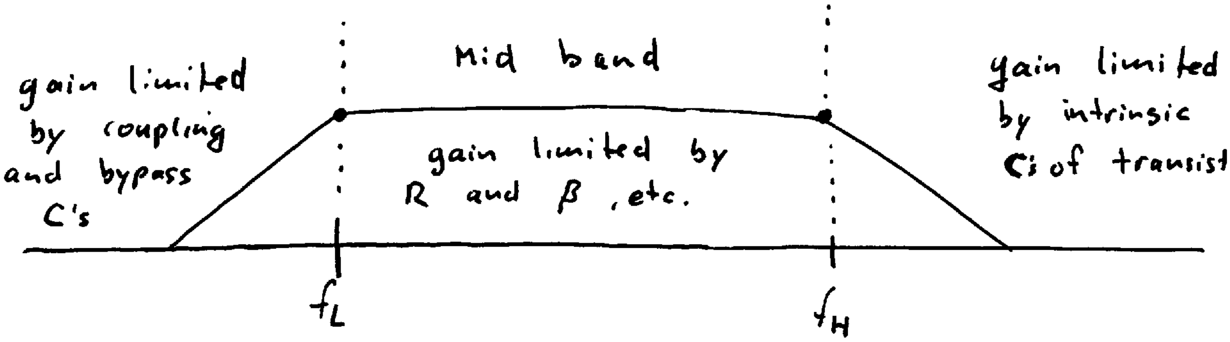
$$f_L \approx \sqrt{f_{L1}^2 + f_{L2}^2 + \dots f_{LN}^2}$$

$$\frac{1}{f_H} \approx \sqrt{\frac{1}{f_{H1}^2} + \frac{1}{f_{H2}^2} + \frac{1}{f_{H3}^2} \cdots \frac{1}{f_{H4}^2}}$$

As can easily be verified these definitions reduce to the previous definitions for well seperated cut-off frequencies.

In every textbook you can find a different approach. They are *all* approximations. It is more important to understand where the cut-off frequencies are coming from – what is limiting the circuit – then to exactly know these frequencies.

First design the circuit, then simulate it with SPICE. Then build it and test it in practice.

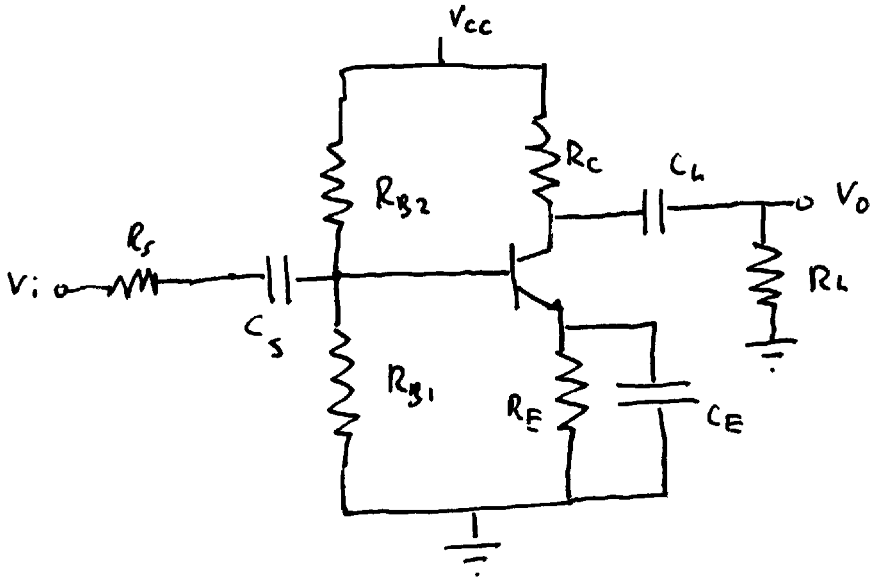


Frequency Response of the common-emitter amplifier :

p608 Sedra

we will now add (more) coupling condensators and a bypass condensator. to our CEA.

The function of a coupling condensator is to remove the DC part of the input or output signal. The function of a bypass condensator is to increase the midband gain, as we will see, maintaining the bias polarization conditions.



C_L, C_S : coupling capacitors

C_E : bypass capacitor.

The capacitors C_S, C_L and C_E each introduce a lower cutoff frequency ; they function as high-pass filters. For C_S and C_L this is easy to see. Remember that a capacitor is short circuit for high frequencies and open-circuit for low frequencies. Why is C_E also a high-pass filter ?

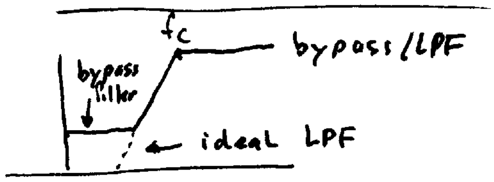
Remember that the gain of a CEA is equal to the resistance at the collector divided by the resistance at the emitter. Thus

$$\boxed{A = \frac{R_c}{R_E + r_e} \quad \text{low frequencies}}$$

and

$$\boxed{A = \frac{R_c}{r_e} \quad \text{high frequencies}}$$

Although this is not really a low-pass filter ($A_{DC} \neq 0$), we can consider it as such.



⟸ ideal LPF has a zero at $f = 0$ Hz and a pole at f_c
bypass filter has a zero at $f \neq 0$ Hz and a pole at f_c

For the low frequency behavior we thus have 3 filters. One determined by C_S, one by C_L and one by C_E. Which one is domination. For C_S, should we consider C_L and C_E as open circuit or close circuit? (It will determine what effective resistance C_S sees and what will be the cut-off frequency.) One trick we can use is the method of

SHORT-CIRCUIT TIME CONSTANTS

TO DETERMINE LOWER FREQUENCY

SHORT CIRCUIT TIME CONSTANTS METHOD

⇒ source to ground

⇒ Consider every other capacitor as short circuit (For instance, for C_S consider C_E and C_L as short circuit).

⇒ Determine $\tau_i = C_i R_{eff,i}$ for every capacitor in this way.

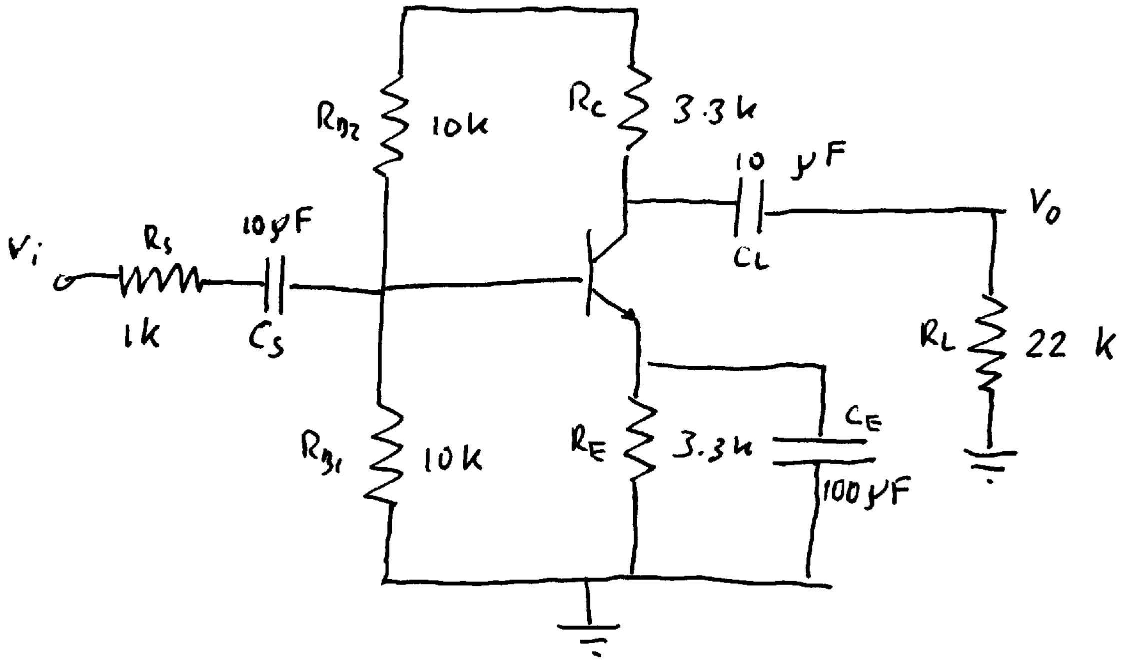
⇒ $\tau_{tot}^{-1} = \sum \frac{1}{\tau_i}$

⇒ $\omega_L = \frac{1}{\tau_{tot}} = \sum_i^N \frac{1}{\tau_i}$

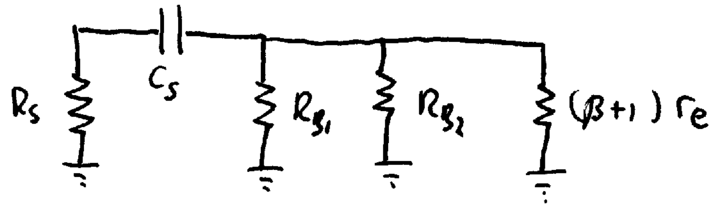
FOR LOW FREQUENCIES

Without proof: while this will not give correct results for the individual time constants, the final result for ω_L is correct (p 690 of Sedra).

As a byproduct, we will immediately see which capacitor is the limiting one.



C_S : τ_S = $10\mu F \cdot R_{eff}$. R_{eff} : short circuit all other C's:

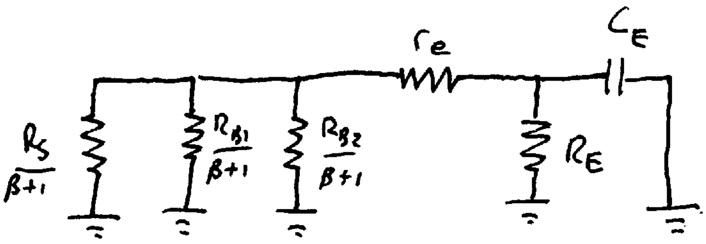


(note : R_E bypassed)

$r_e = 20\ \Omega$
$r_\pi = 2\ k\Omega$

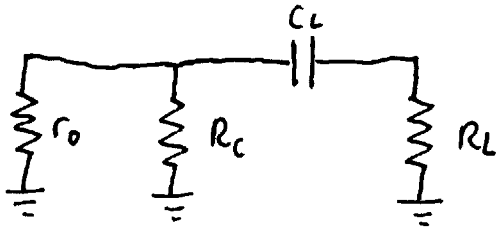
$$R_{eff} = R_S + (R_{B1} // R_{B2} // r_\pi)$$
$$= 1k\Omega + (10k\Omega // 10k\Omega // 2k\Omega)$$
$$= 2.43\ k\Omega$$
$$\tau_S = 10\mu F \times 2.43\ k\Omega = 24\ ms \quad (f_{LS} = \frac{1}{2\pi\tau} = 6.5\ Hz)$$

C_E : $\tau_E = 100\ \mu F \cdot R_{eff}$



$$R_{eff} = R_E // (r_e + (R_{B1} // R_{B2} // R_S)/\beta+1)$$
$$= 3.3\ k\Omega // (20\ \Omega + (10k\Omega // 10k\Omega // 1k\Omega)/101)$$
$$= 3.3\ k\Omega // (20\ \Omega + 8.3\ \Omega)$$
$$= 28\ \Omega$$
$$\tau_E = 100\ \mu F \times 28\ \Omega = 2.81\ ms \quad (f_{LE} = \frac{1}{2\pi\tau} = 56.7\ Hz)$$

C_L . $\tau_L = 10\ \mu F \times R_{eff}$



$$r_o \approx \frac{V_A}{I_c} \approx 200\ k\Omega$$

$$R_{eff} = (r_o // R_c) + R_L$$

$$= (200\ k\Omega\ // \ 3.3\ k\Omega) + 22\ k\Omega$$

$$= 25.2\ k\Omega$$

$$\tau_L = 10\ \mu F \cdot 25.2\ k\Omega = 0.25\ s \quad \left(f_{LL} = \frac{1}{2\pi \tau_L} = 0.63\ Hz \right)$$

---

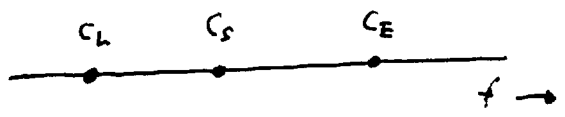
conclusions

① $$\tau_{tot}^{-1} = \sum_{i=1}^{N} \frac{1}{\tau_i} = \overset{\overset{C_E}{\Uparrow}}{\frac{1}{2.81\ ms}} + \overset{\overset{C_S}{\Uparrow}}{\frac{1}{24\ ms}} + \overset{\overset{C_L}{\Uparrow}}{\frac{1}{250\ ms}} \Rightarrow$$

$$\tau_{tot} = 2.5\ ms$$

$$\omega_L = \frac{1}{\tau_{tot}} = 400\ rad/s\ , \quad f_L = \frac{\omega_L}{2\pi} = 64\ Hz$$

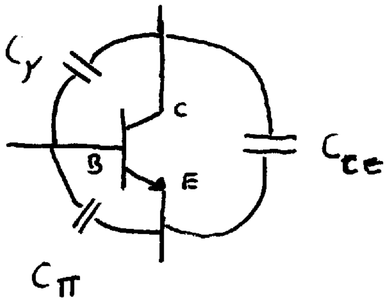
② The bandwidth is limited by the capacitor giving the shortest time constant : C_E.
If we want to increase the bandwidth, we should increase C_E or decrease β (different transistor) or increase R_s.



High - frequency analysis

The high frequency response is normally limited by the intrinsic capacitances of the transistors.
These capacitances are unavoidable and have a physical nature (see my option lectures "Physics of semiconductor

devices"). We can find the values for the capacitances in the data sheets of the transistors used



capacitances of an npn transistor

C_μ is capacitance between collector and base

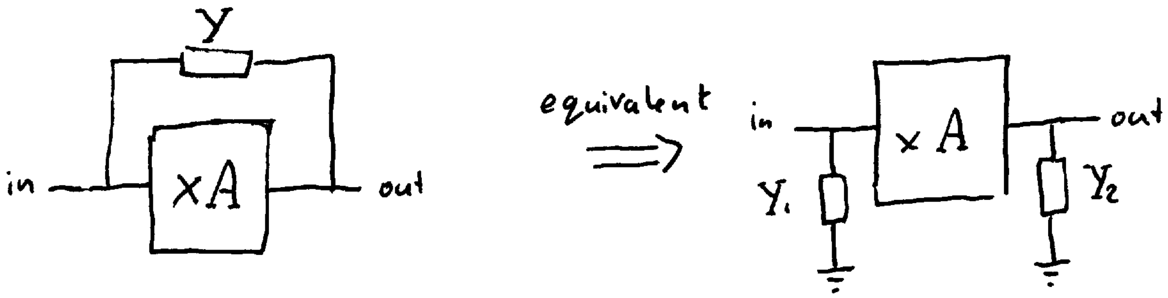
C_π is capacitance between base and emittor

C_{ce} is capacitance between collector and emittor and is normally emitted from the calculations.

Examples: $C_\pi = 25$ pF, $C_\mu = 10$ pF

To find the upper cut-off frequency f_H we have to find the effective resistance these capacitors see. However, there is a complication: The capacitors are connected to both the input and the output. For instance C_μ is at the input (V_B) and the output (V_C) of the transistor. This causes feedback (see next chapter). We can analyze it if we use Miller's theorem

Miller's Theorem



An admittance Y (for instance C, L or 1/R) bridging part of a circuit with voltage gain A can be decomposed into a circuit with an admittance at the entrance (Y_1) and at the exit (Y_2) of the amplifier.

$$\boxed{\begin{aligned} Y_1 &= Y(1-A) \\ Y_2 &= Y(1-1/A) \end{aligned}}$$

The admittance is amplified at the entrance and reduced (slightly) at the exit.

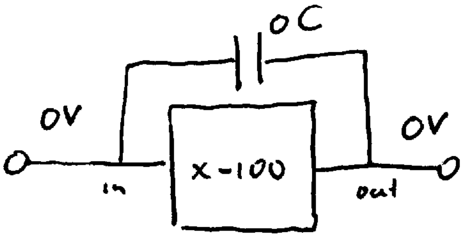
How can this be? Take the example of a capacitor C bridging an amplifier with 100 x gain. The capacitance that is felt at the entrance is the charge that is stored in the capacitor. By definition:

$$C_{eff} = \frac{dq}{dV}$$

example $C = 1nF$

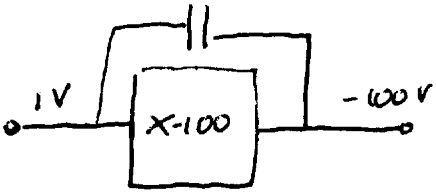
$A = -100.$

At 0 V at entrance: the exit voltage is also 0 V and the capacitor is empty



$$Q = \Delta V C = (V_{in} - V_{out}) \times 1nF = 0\ C.$$

Let us increase it to 1 V at the entrance and see how much charge will be in the capacitor. with a gain of -100 there will be -100 V at the other side. The amount of charge in the capacitor is then

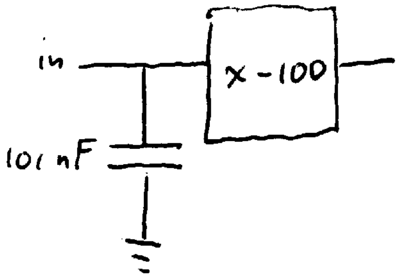


$$Q = \Delta V \cdot C = (1-(-100)) \times 1nF = 101\ nC$$

Seen from the entrance, it looks like a capacitor

$$C = \frac{\overset{\text{total charge}}{Q}}{\underset{\text{input signal}}{1V}} = \frac{101\ nC}{1V} = 101\ nF$$

in other words, at the input it looks like the figure below.



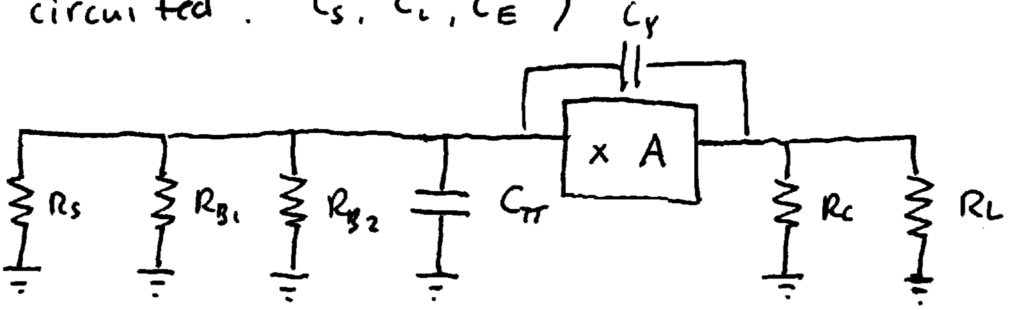
For other types of admittance we can make a similar admittance. In summary:

capacitances are multiplied at entrance. Resistances are reduced

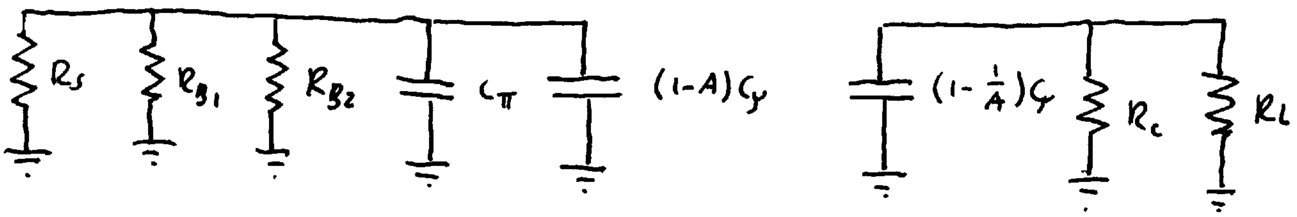
Note that we can only use Millers theorem when the admittance does not (significantly) change the gain. It can (therefore) also not be used to determine the input and output resistances of the amplifier.

The effect on the capacitances is called the Miller effect and this influences the high-frequency response of the amplifier.

We will consider the two capacitors C_μ and C_π.
They will cauce low-pass filters
As the circuit we have (note: all other capacitors short circuited. C_s, C_L, C_E)



⇓ MILLER



The voltage gain A from the entrance to the exit of the transistor (base - to - collector, where the capacitor is connected) is (note: R_E is bypassed)

$$A = \frac{-(R_C // R_L)}{r_e} = -\frac{(3.3\,k\Omega // 22\,k\Omega)}{20\,\Omega} = -143$$

(note the tremendous effect in gain caused by bypassing R_E. From ≈ -1 to ≈ -143)

Now we can calculate the time constants:

$$\tau_{in} = (R_S // R_{B1} // R_{B2})\,(C_\pi // (1-A)\,C_y)$$

$$= (1\,k\Omega // 10\,k\Omega // 10\,k\Omega)\,(25\,pF + 144 \cdot 10\,pF)$$

$$= 833\,\Omega \cdot 1465\ 10^{-12}\ F$$

$$= 1220\ ns \qquad (f_{H\,in} = \frac{1}{2\pi\,\tau_{in}} = 130\ kHz)$$

[note: the "Miller effect" for C_π is 1, because the "gain" from base to emitter is 0 (emittor is connected to ground), thus $C_M = (1-A)\,C_\pi = (1-0)\,C_\pi = C_\pi$]

$$\tau_{out} = (R_C // R_L)\,\left((1-\tfrac{1}{A}) \cdot C_y\right)$$

$$= (3.3\,k\Omega // 22\,k\Omega)\,(1 + \tfrac{1}{143}) \cdot 10\,pF$$

$$= 2.87\,k\Omega \cdot 10.07\,pF$$

$$= 29\ ns \qquad (f_{H\,out} = \frac{1}{2\pi\,\tau_{out}} = 5.5\ MHz)$$

There are thus two low-pass filters. One with a time constant of 1220 ns and the other with

$\tau = 29$ ns. It is immediately clear (the time constants are well seperated) that the first one is dominant. The cut-off frequency thus becomes

$$\omega_H = \frac{1}{\tau_{in}} = 8.2 \; 10^6 \text{ rad/s}$$

$$f_H = \frac{\omega_H}{2\pi} = 130 \text{ kHz}.$$

In case the individual cut-off frequencies are not well seperated, we can use the trick of

OPEN-CIRCUIT TIME CONSTANTS TO DETERMINE HIGHER CUT-OFF FREQUENCY

- ⇒ source to ground
- ⇒ for every capacitor causing a LPF, consider all other capacitors (of LPF's) as open circuit.
- ⇒ Determine τ_i for every capacitor in this way
- ⇒ $\tau_{tot} = \sum \tau_i$
- ⇒ $\omega_H = \frac{1}{\tau_{tot}} = \frac{1}{\sum \tau_i}$, $f_H = \frac{1}{2\pi} \omega_H$

Like for lower cut-off frequency, this is exact

In this case we would find

$$\tau_{tot} = \tau_{in} + \tau_{out} = 1220 \text{ ns} + 29 \text{ ns} = 1249 \text{ ns}$$

$$\omega_H = \frac{1}{1249 \text{ ns}} = 8.0 \; 10^6 \text{ rad/s}$$

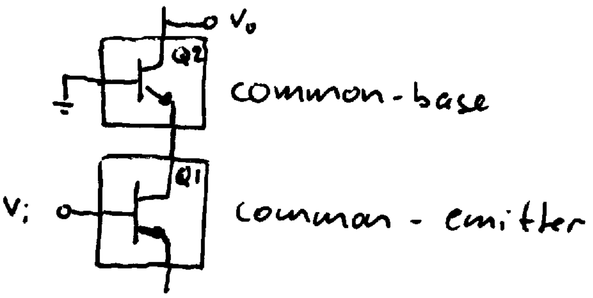
$$f_H = 127 \text{ kHz}$$ (compare to dominat-pole technique)

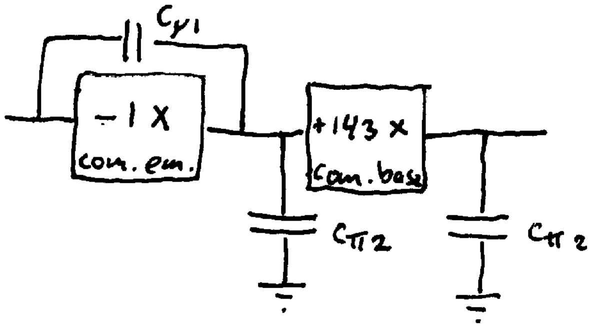
It is clear that the response is completely limited by the Miller effect. Without it we would win more-or less two decades (130 kHz to 13 MHz).

⇒ Insert p 22

As we will see in the practical lectures, the sollution to this is the cascode amplifier.

A cascode amplifier is a common-emitter amplifier in series with a common-base amplifier

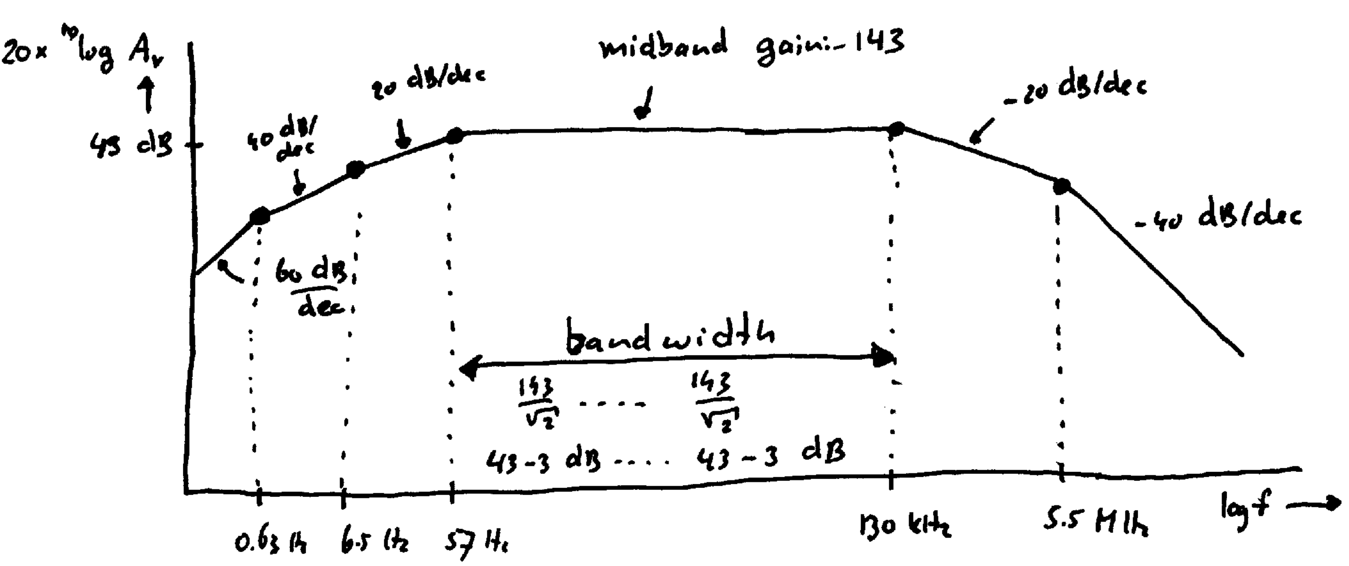


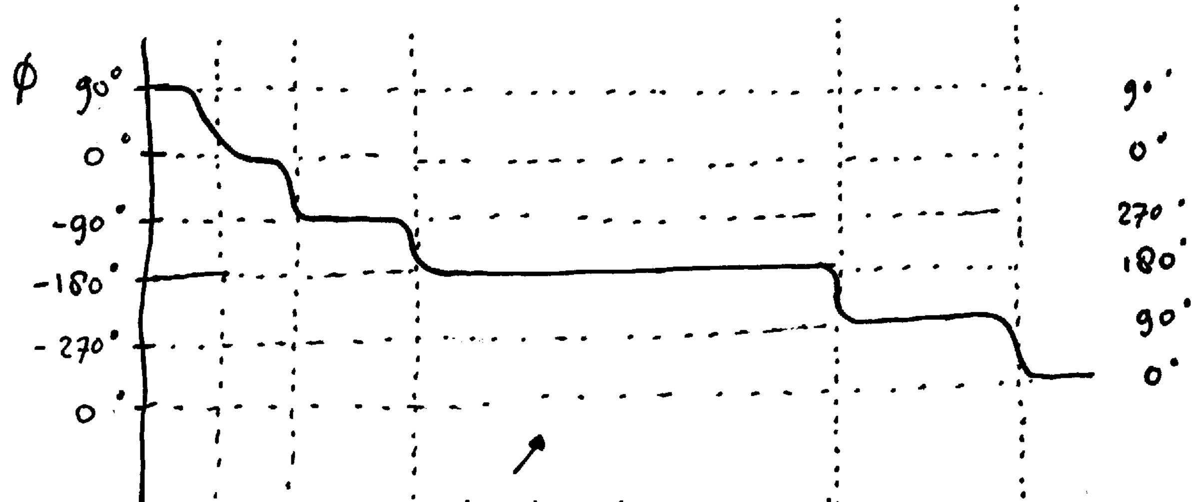


The gain of the common emitter is −1, because it is $-\frac{r_{e2}}{r_{e1}}$. The Miller effect is reduced to only a factor $1-(-1) = 2$.

The common-base stage has a high gain $\left(+\frac{R_c // R_L}{r_{e2}}\right)$ but because the base is connected to ground, the Miller effect has disappeared (= 1). Only $C_{\pi 2}$ at input and output.

Summary / Bode plot of the CEA





midband gain is negative
⇒ phase 180° (-180°)

At the practical lessons we will further study the frequency response of the differential pair of chapter 1. (For more information, see Sedra's chapter 7.8)

CHAPTER 3

FEEDBACK

ch. 8 of Sedra

Feedback is the concept of putting part of the output signal back at the entrance input of the amplifier or circuit.

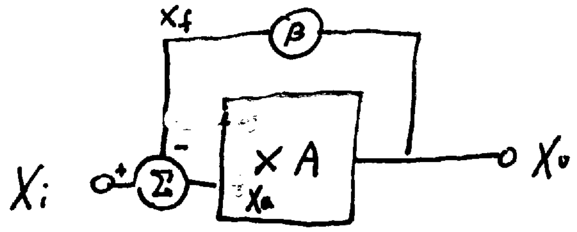
There are various advantages to this :

- stabilize the gain (independence on β, etc)
- change output and input resistances
- extend the bandwith

There are also negative effects such as

- instability (oscillations)

The basic feedback circuit consists of an amplifier, with a gain A and a feedback loop, which put part, β, of the output back at the input. For negative feedback this is subtracted at the input :



(We consider here only negative feedback. It is not so difficult to see that positive feedback results in a runaway signal).

For negative feedback like in the figure above, it can be shown that

(1) $X_o = A X_a$ (X can be voltage or current !)

(2) $x_f = \beta x_o$

(3) $x_a = x_i - \cancel{R}_f = x_i - \beta x_o$

(3) into (1) :

$$x_o = A(x_i - \beta x_o)$$

$$\boxed{\frac{x_o}{x_i} = \frac{A}{1 + A\beta}}$$ ⟸ NEGATIVE FEEDBACK

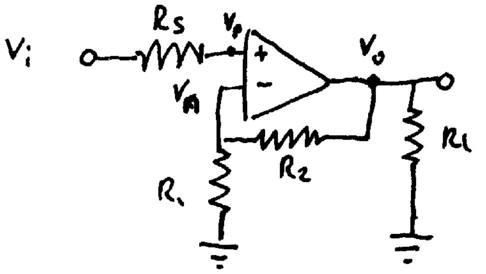
$A\beta$ is called the loopgain

When the amplifier A is an ideal amplifier, $A = \infty$ and the amplification becomes

$$\frac{x_o}{x_i} = \frac{1}{\beta}$$

This is an interesting result; the gain of an ideal amplifier with feedback is determined by the feedback loop.

Operational amplifiers have very large gain (of the order of 10^5) and the gain of op-amp's is therefore controlled by the feedback. As an example



op-amp: $A = 10^4$

$r_{in} = \infty$

$r_{out} = 0$

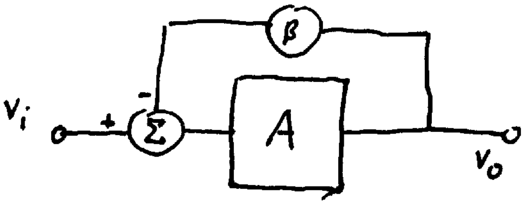
$R_2 = 10\,k\Omega$

$R_1 = 1\,k\Omega$

- The feedback is from V_o to V_n and is supplied by a voltage divider R_1, R_2:

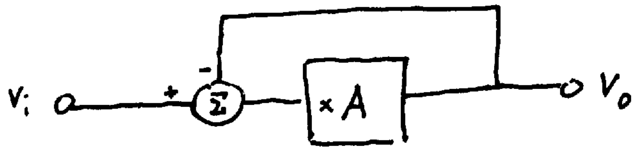
$$\beta = \frac{R_1}{R_1 + R_2} = \frac{1\,k\Omega}{1k\Omega + 10\,k\Omega} =$$

- The open-loop gain is the gain from v_i to V_o, with the feedback disconnected. This is thus 10^4
- Because $r_{in} = \infty$, $i_i = 0$ and $V_p = V_i$



$$\left.\begin{aligned} A &= 10^4 \\ \beta &= \end{aligned}\right\} \quad \frac{V_o}{V_i} = \frac{A}{1 + A\beta} =$$

In the other extreme case, when the amount of feedback is 100%

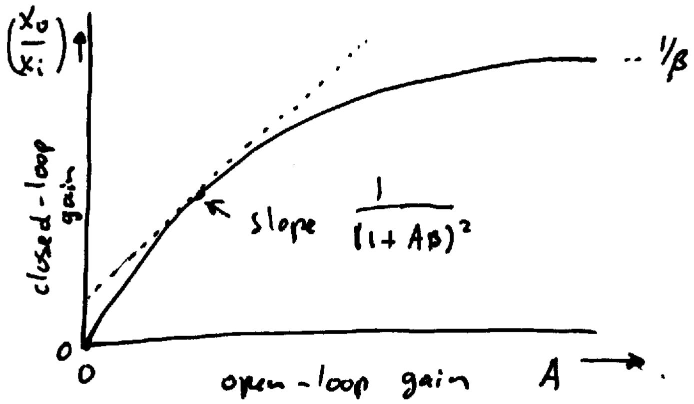


In this case the gain is

$$\frac{v_o}{v_i} = \frac{A}{1 + A\beta} = \frac{A}{1+A} \approx 1$$

The voltage at the output is exactly the voltage at the input. For this it is called a tension-follower the advantage is that it can convert a circuit with high output resistance into a circuit with low output resistance. (Imagine connecting 8 Ω speakers to our differential pair).

Desensitizing the gain. Not all amplifiers (op amps) coming from the factory are equal. They all have slightly different gain. Typically in the order of 5%. For some applications this is not accurate enough. The gain can be stabilized by feedback



$$\frac{d\left(\frac{X_o}{X_i}\right)}{dA} = \frac{1}{(1+A\beta)^2}$$

$$\frac{d\left(\frac{X_o}{X_i}\right)}{dA\left(\frac{X_o}{X_i}\right)} = \frac{1}{(1+A\beta)^2} \Big/ \frac{A}{(1+A\beta)} .$$

The relative change in gain is thus related to the relative change in gain of the open circuit :

$$\frac{d\left(\frac{X_o}{X_i}\right)}{\left(\frac{X_o}{X_i}\right)} = \frac{dA}{A} \times \frac{1}{(1+A\beta)} .$$

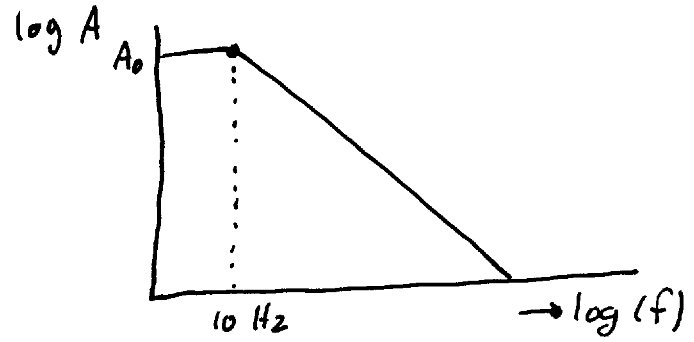
Thus, the effect of variations of different amplifiers is reduced by a factor $\frac{1}{1+A\beta}$. The gain is much more stable than the open circuit amplifier. an "error" in gain of the amplifier of $\frac{dA}{A} = 5\%$ has only a relative error of $\frac{1}{1+A\beta} \cdot 5\%$ in the final circuit.

The price that is payed is a reduced gain :

From A to $\frac{A}{1+A\beta}$

Bandwidth extension

An amplifier with feedback has an increased bandwidth. As an example, take a typical op-amp.



with a cut-off frequency at 10 Hz. The transfer function of this is

$$A(s) = \frac{A_0}{1 + s/\omega_0}$$

$\omega_0 = 2\pi \times 10$ Hz
$A_0 = 10^5$ (typical)

using this op-amp in an amplifier with feedback β:

$$A_f(s) = \frac{A(s)}{1 + A(s)\beta} = \frac{A_0/(1+s/\omega_0)}{1 + A_0\beta/(1+s/\omega_0)}$$

$$= \frac{A_0/(A_0\beta+1)}{1 + s/(\omega_0\ (1+A_0\beta))}$$

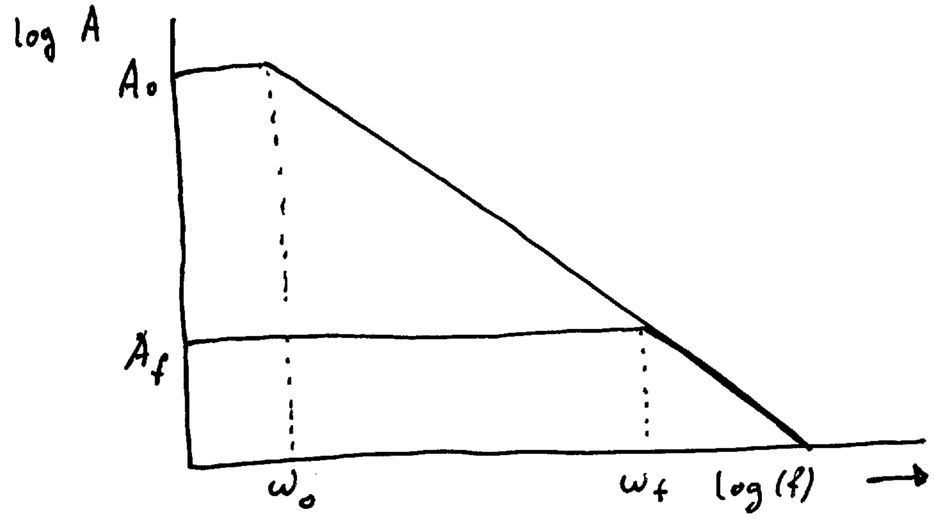
In other words:

The DC gain $= A_0/(A_0\beta+1)$

and the new cut-off frequency is

$$\omega_f = \omega_0 \, (1 + A_0 \beta)$$

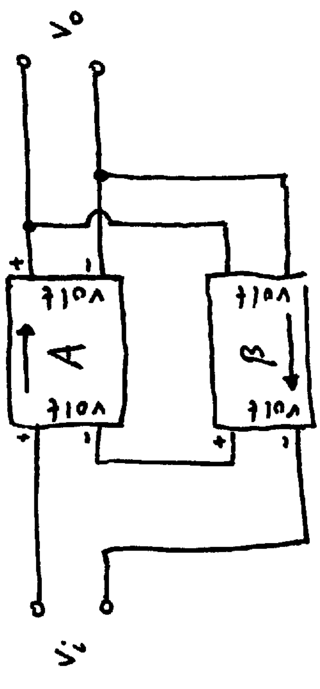
By using feedback, the bandwidth of the amplifier is increased. The price that is payed is a reduced midband gain $A_0 \rightarrow A_f = \frac{A_0}{1 + A_0 \beta}$



Topologies

In the examples we used above, the output of the amplifier was a voltage signal and this was partially fed-back with a factor β and added to the input as a voltage signal. This topology is called voltage-sampling series-mixing (A)

A) SERIES - SHUNT

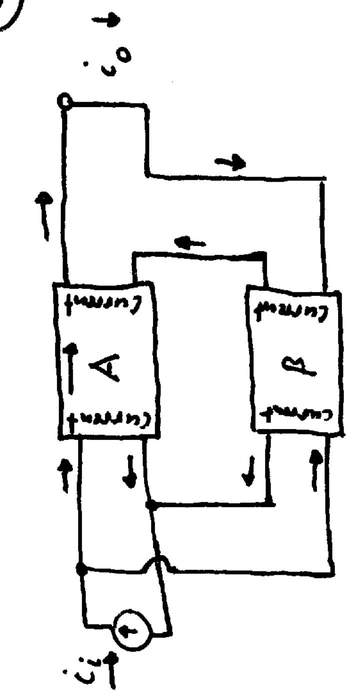


a) voltage-Sampling Series mixing

Amplifier has voltage amplif.
(input : V , output : V)

The feed back loop β also has
input: V , output : V
and is subtracted at input (series)

B) SHUNT - SERIES

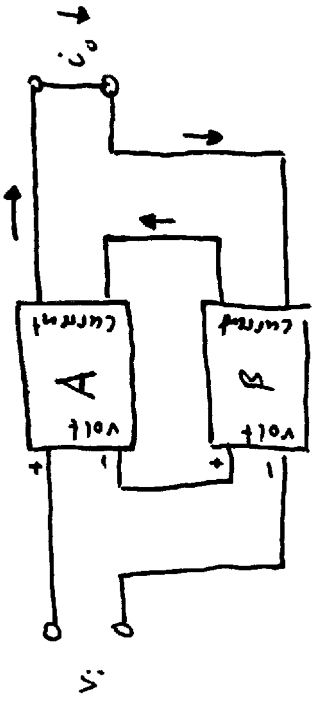


b) Current Sampling Shunt-mixing

Amplifier has current amplific.:
input : i , output : i

The feedback loop samples current
input: i , output : i
and is added to input.

C) SERIES-SERIES

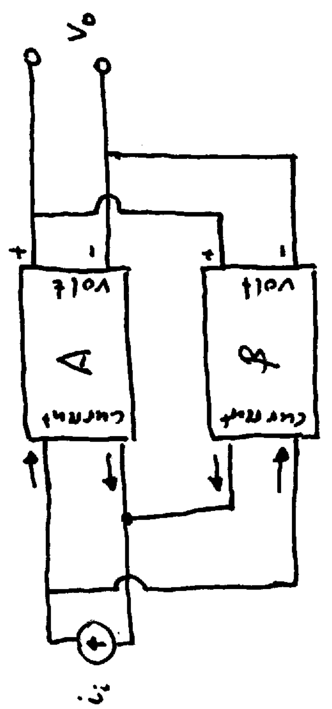


c) Current Sampling Series mixing

Amplifier is transconductance ampl.:
input : v , output : i

Feedback loop has
input : i , output : v
v is subtracted at input of A

D) SHUNT - SHUNT



d) voltage Sampling shunt-mixing

Amplifier is transresistance amplifier
input : i, output : v

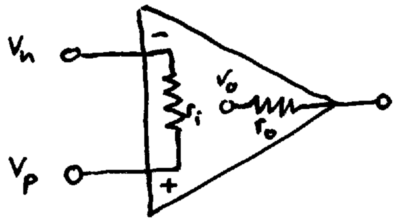
Feedback loop β has
input : v , output : i
i is subtracted at input of A

The other extreme is where the amplifier is a current amplifier. This current is sampled and with a factor β subtracted at the input (B)

The other two topologies are mixtures of current and voltage. (C) has a transconductance amplifier (v → i)

Output Resistance

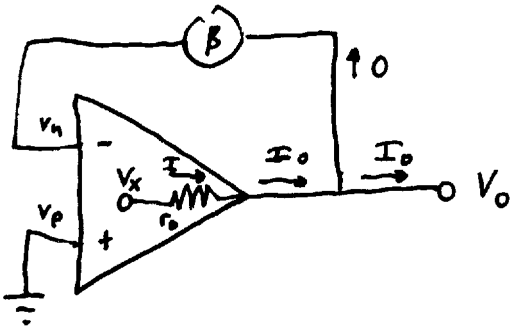
The input and output resistances of the amplifier also change significantly. This is best illustrated on basis of an operational amplifier



r_i : input resistance

r_o : output resistance

To determine the effect on the output resistance we consider the input resistance infinite and connect (for the non inverting amplifier) V_p connected to ground.



(Because $r_i = \infty$ no current goes to v_n)

$$V_x = -A v_n$$

$$V_o = V_x - I_o r_o$$

$$V_n = \beta V_o$$

substituting gives

$$I_o = V_o \frac{1 + A\beta}{r_o}$$

with the definition of output resistance

$$r_o \equiv 1 / \frac{\partial I_o}{\partial V_o}$$ we get

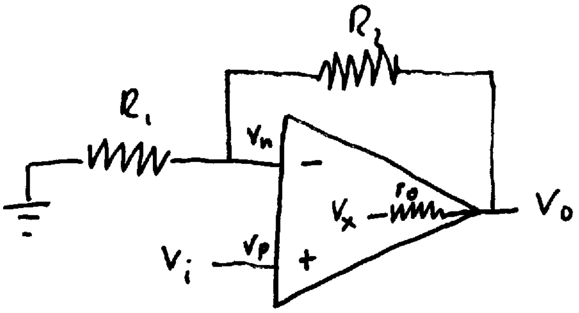
$$\boxed{r_{of} = \frac{r_o}{1 + A\beta}}$$

In other words : the output resistance is reduced by a factor $1 + A\beta$ by using feedback.

Since the amplifier is a linear circuit, for other voltages at the input (V_p) we will find the same result.

In the above example a feedback that is not loading the output was assumed. Normally the feedback loop is made with resistances and other conductive elements. In this case, there is a current being drawn from the output of the amplifier. When this amplifier has zero output resistance, the effect is nil; the amplifier can easily supply the current. If the output resistance r_o of the amplifier is not zero there is an effect on the open loop gain A.

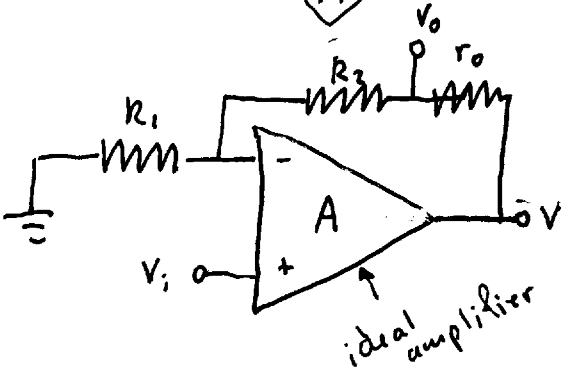
As an example : assume $r_o \neq 0$:



$$\beta_0 = \frac{R_1}{R_1 + R_2}$$

$$\frac{V_o}{V_i} = \frac{A}{1 + A\beta_0} \leftarrow \text{ideally } (r_o = 0)$$

$$\Downarrow$$



$$\frac{V_x}{V_i} = \frac{A}{1 + A\beta'} \quad , \quad \beta' = \frac{R_1}{R_1 + R_2 + r_o}$$

$$\frac{V_o}{V_i} = \frac{V_x}{V_i} \cdot \frac{V_o}{V_x} = \frac{A}{1 + \frac{R_1}{R_1 + R_2 + r_o} \cdot A} \cdot \frac{R_1 + R_2}{R_1 + R_2 + r_o}$$

$$= \frac{A\alpha}{1 + A\alpha\beta_o} \quad , \quad \text{with } \alpha = \frac{R_1 + R_2}{R_1 + R_2 + r_o}$$

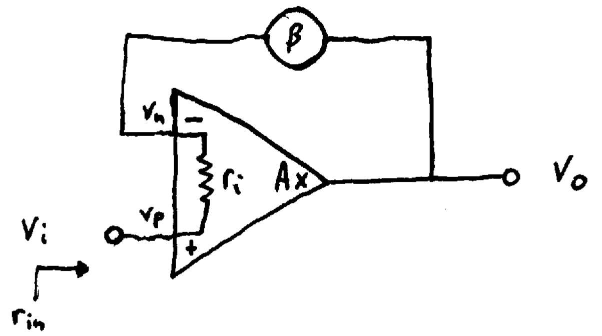
$$\beta_o = \frac{R_1}{R_1 + R_2}$$

In other words: because of the current loading of the feedback loop ($R_1 + R_2 < \infty$) and the non-zero output resistance ($r_o \neq 0$) the open-loop gain is reduced to

$$\boxed{A_1 = A \cdot \frac{R_1 + R_2}{R_1 + R_2 + r_o}}$$

Input Resistance

The input resistance can significantly increase when feedback is used



In this case it can be shown that :

$$V_o = (V_p - V_n)A = (V_i - V_n)A$$

$$V_n = \beta V_o = A\beta(V_i - V_n)$$

$$V_n = \frac{A\beta}{1+A\beta} V_i$$

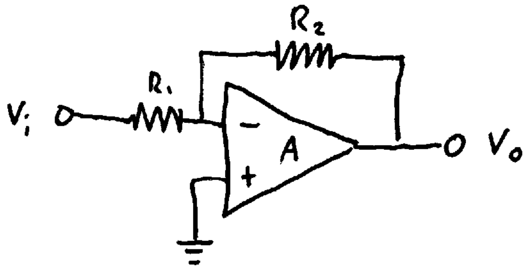
$$I_i = \frac{V_p - V_n}{r_i} = \frac{V_i - \frac{A\beta}{1+A\beta}V_i}{r_i} = \frac{1}{r_i} \cdot \frac{1}{1+A\beta} \cdot V_i$$

$$\boxed{r_{in} \equiv 1 / \frac{\partial I_i}{\partial V_i} = r_i\,(1+A\beta)}$$

Because of the feedback, the input resistance of this amplifier has increased a factor $1+A\beta$.

Not always is the increase in input resistance so dramatic

Example : Inverting amplifier with negative feedback.



a) what is the gain ($A_v = \frac{V_o}{V_i}$) of this circuit ?

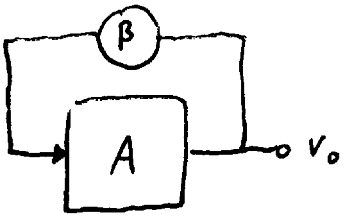
b) what is the output resistance ?

c) what is the input resistance ?

(Answ: $-\frac{AR_2}{AR_1 + (R_1+R_2)}$, $r_{out} = \frac{r_o}{1+A\beta}$, $r_{in} = R_1 + \frac{R_2}{1+A}$)

Feedback and Stability / oscillators

One of the negative aspects of feedback is that it can induce instabillities (oscillations) in the signals. This means that we can have an output signal at a certain frequency without having any input signal connected. It is clear that passive circuits (without amplification in any point) cannot oscillate, because the energy needed has to come from somewhere.



Generally speaking, a circuit with feedback β and open-loop gain A will oscillate when

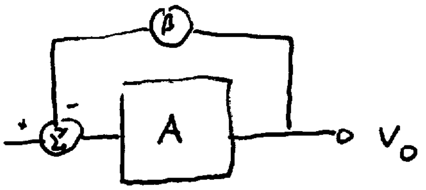
$\underline{A\beta = 1}$. This is the $\boxed{\text{Barkhausen}}$ $\boxed{\text{Criterion}}$

For this case, any voltage at input of amplifier is added with factor 1 at amplifier and added again and again ... ab infinito! The output signal will be infinite. Note that often A and β depend on frequency. So it can occur that the output will be infinite for only certain frequencies

It will oscillate for all frequencies at which $A(f)\cdot\beta(f) = 1$

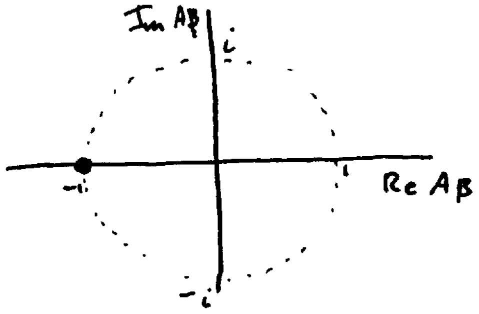
For our amplifier with negative feedback we found a gain of

$$\frac{X_o}{X_i} = \frac{A}{1+A\beta}$$



Also here we can see that if $A\beta = -1$ then the output will be infinite for any tiny voltage at input. Normally, noise is present at any point of the circuit (~ μV .. mV) and this will initiate the oscillations.

$A\beta = -1$ is equal to saying $|A\beta| = 1$ and the phase $\angle A\beta = 180°$, or in a phase diagram

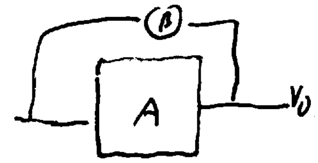


⇐ The ● gives the Barkhausen Criterion for amplifiers with negative feedback

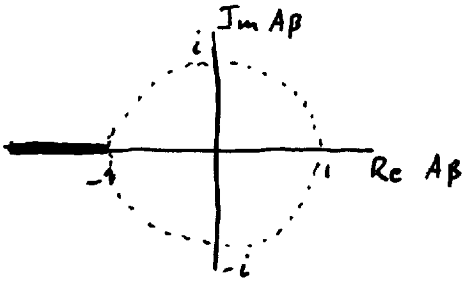
The --- represents the points where $|A\beta| = 1$.

---

using the same reasoning (positive feedback): If the loop gain $A\beta > 1$, also oscillation will occur. For instance, for $A\beta = 2$ and starting with 1 μV at the entrance, this voltage is fed back to the entrance with a factor 2 and added. Now we have 3 μV at entrance. Then 3 + 6 μV = 9 μV ... etc.



In our picture of negative feedback :

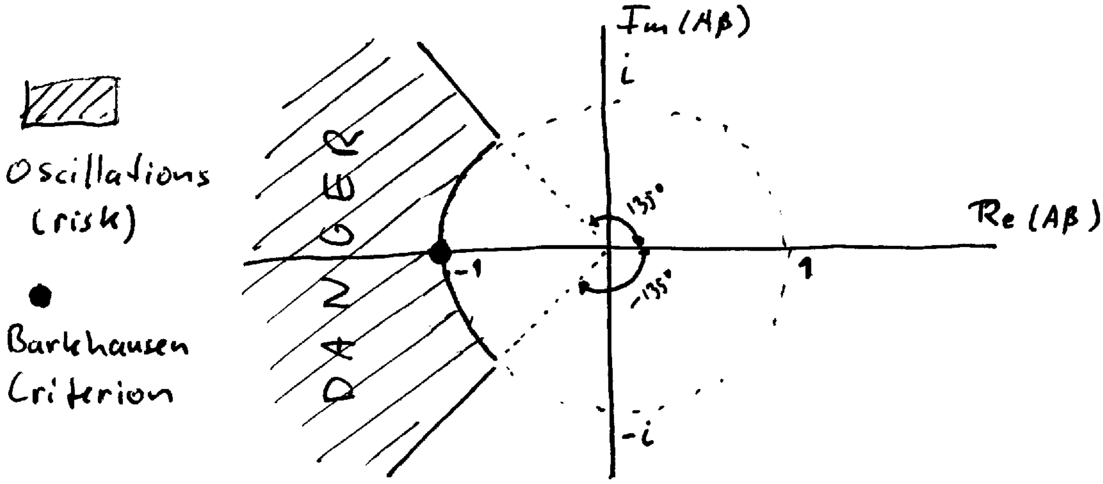


Oscillations when $|A\beta| > 1$

$\angle A\beta = 180°$

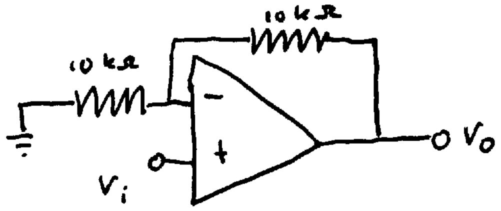
see ▬

Finally, a good electronic designing practice is to allow for a phase margin in the design. Normally 45° is used. With this, the final Nyquist plot (Im (Aβ) v.s. Re (Aβ)) becomes



Negative feedback. Nyquist plot.

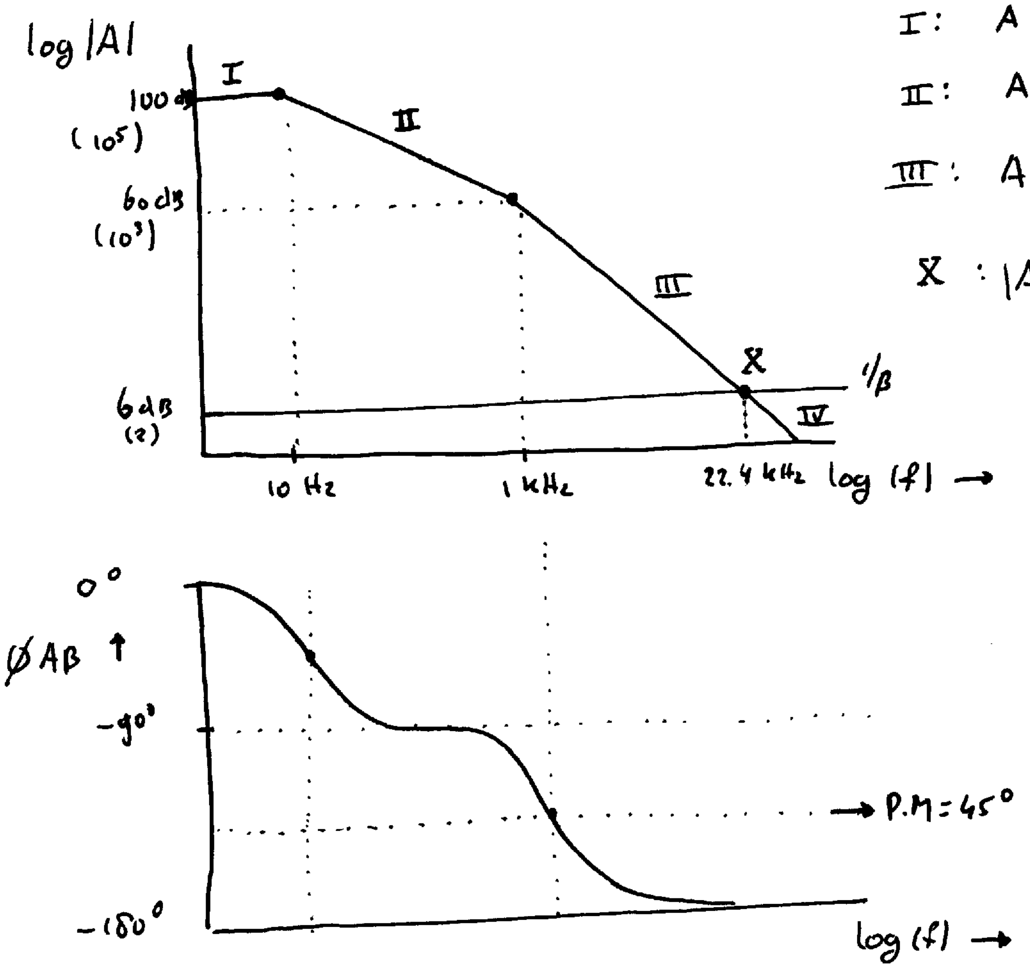
As an example :



An op-amp with negative feedback. A has two poles, 10 Hz and 1 kHz. The DC gain is $+10^5$. Will this circuit oscillate? If so, at what frequencies? Assume $r_{in} = \infty$, $r_o = 0$, P.M. = 45°

$\beta = \frac{R_1}{R_1+R_2} = 0.5$ $\quad \phi = 0° \Rightarrow \phi(A\beta) = \phi(A)$

$1/\beta = 2$



I: $A = 10^5$

II: $A = 10^5 \cdot \frac{10\,Hz}{f}$

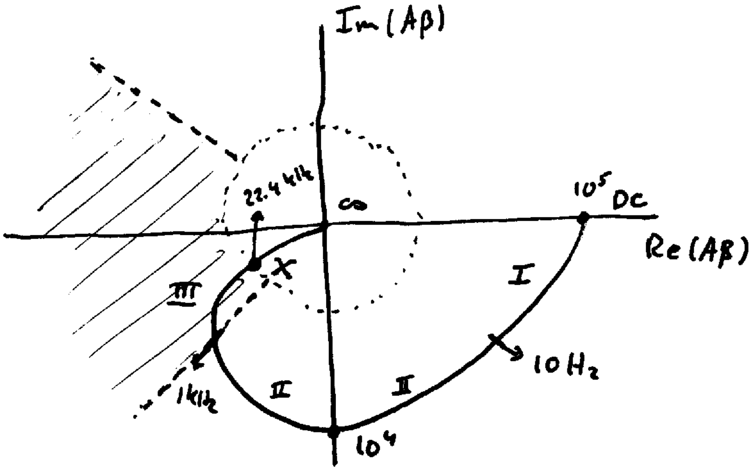
III: $A = 10^3 \cdot \frac{(1\,kHz)^2}{f^2}$

X : $|A| = 1/\beta \Rightarrow |A\beta| = 1$

- When curve of $|A|$ crosses curve of $|1/\beta|$: $|A\beta| = 1$. we are somewhere on the circle in the Nyquist plot. But where ?
- when $|A| < |1/\beta|$, the we are inside the circle and here the circuit is stable . zone IV
- In zones I and II, the phase is between 0° and -135° and the circuit is stable
- The problem is in zone III. Here the loop gain $|A\beta|$ is still larger than 1 and the phase is

inside the danger zone.

In a Nyquist plot:

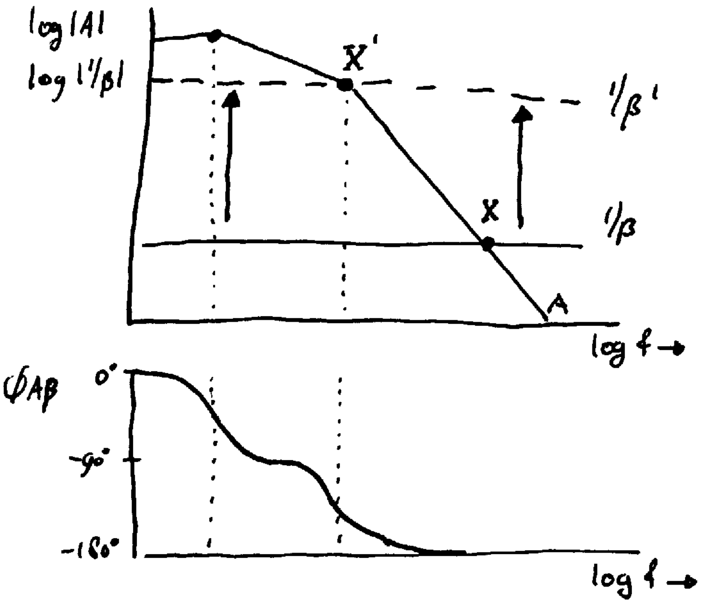


conclusion: The circuit runs the risk of oscillating in the frequency range 1 kHz — 22.4 kHz

The oscillations caused by feedback are not always bad. In fact, we can make oscillators in which the frequency is designed

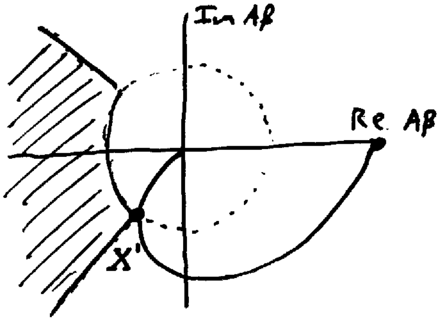
How can the above circuit be stabilized?

1) we could reduce the amount of feedback β



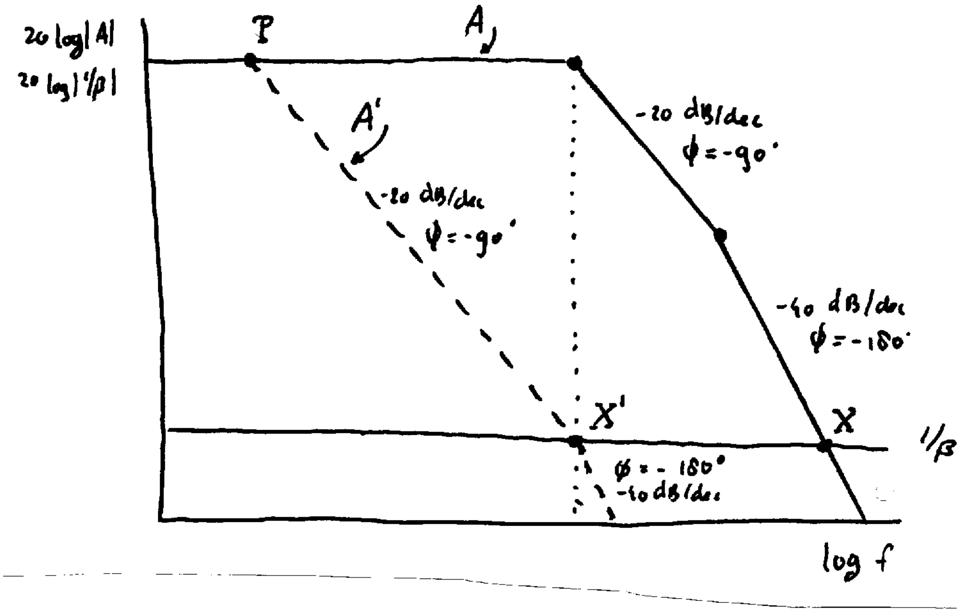
Marginally stable:

X': $|A\beta| = 1$, $\phi A\beta = -135°$.
(P.M = 45°)



It is not difficult to show that a $\beta = 10^{-3}$ is needed to stabilize the circuit. For example with $R_1 = 100\,\Omega$ and $R_2 = 100\,k\Omega$.

2) <u>Frequency compensation</u> is another method for stabilizing the circuit, although it is rather done in the factory of the opamps. It consists of introducing a new pole in the open-loop gain A.



In the above figure: by introducing a new pole P at low frequencies the gain is reduced to below 1/β before the phase changes of the other poles take effect. To be sure this works for any β, a β of 1 has to be used in the calculation for the design of P

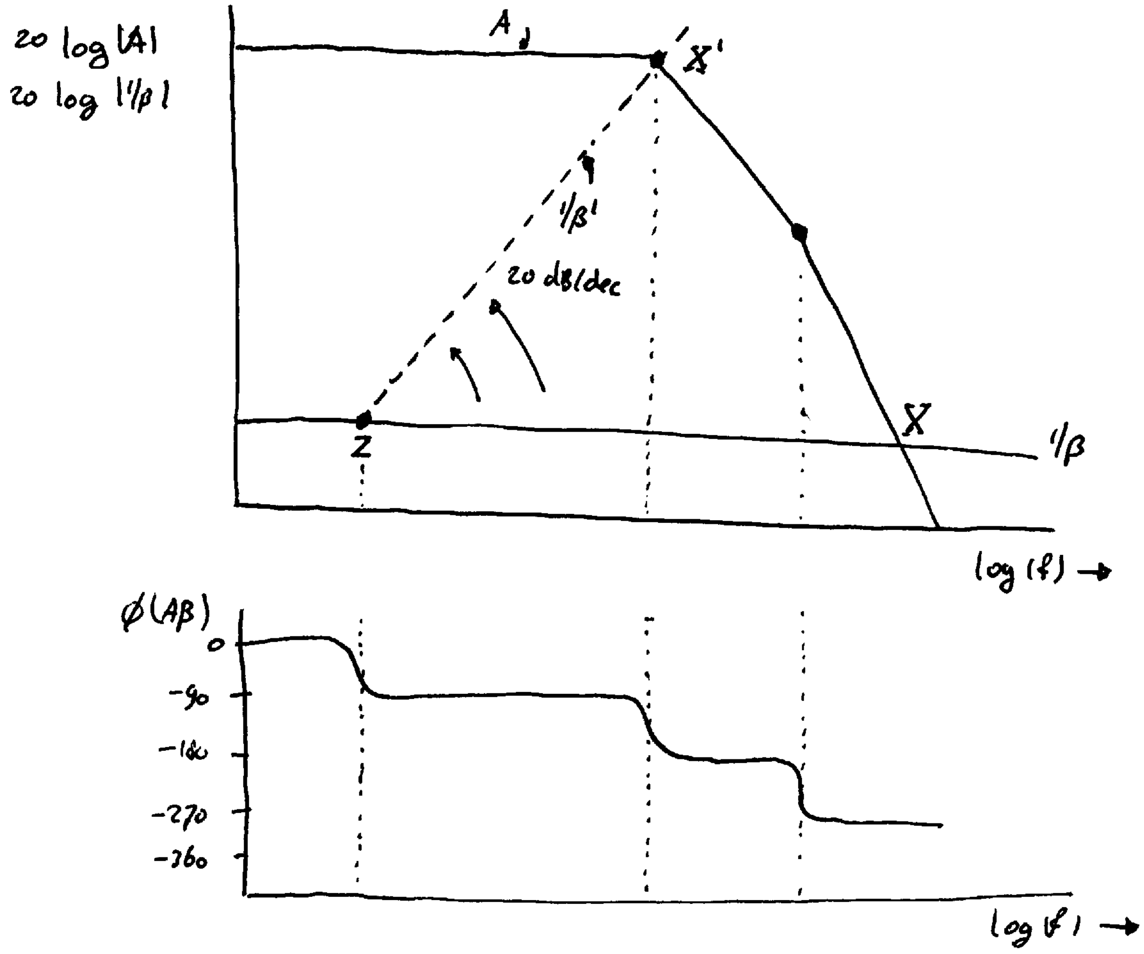
Most modern op-amps implement this technique and have extremely low cut-off frequencies (at around 10 Hz).

Question: what would be the frequency of the pole P needed in our op-amp in order to stabilize it for $\beta = 0.5$ and for $\beta = 1$

(answers: 2×10^{-4} Hz and 1×10^{-4} Hz)

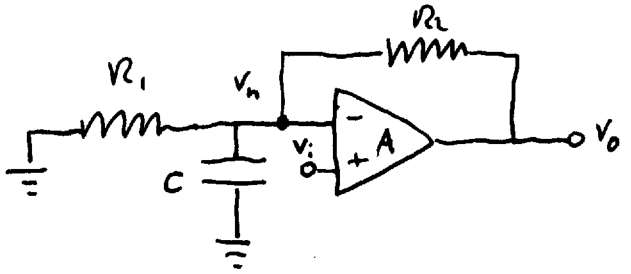
3 External compensation

In a similar way, introducing a pole in β (a zero in $1/\beta$) will result in stabilization of the circuit



By the zero z, the $1/\beta$ is pushed up to make it cross with A earlier, before the first pole of A.

The idea is to put an LPF in the feedback circuit β. For instance:



It is not difficult to see that

$$\beta \equiv \frac{v_n}{v_o} = \frac{R_1 / (R_1+R_2)}{1 + (R_1 // R_2)\, s\, C} \leftarrow \text{DC gain} = \frac{R_1}{R_1+R_2}$$

cut-off frequency

$$\frac{1}{2\pi\, (R_1//R_2)\, C}$$

which we could have directly seen by considering the effective resistance seen by C, with v_o and v_i connected to ground.

Question: what will be the capacitance C needed to stabilize our circuit of page 16?

(Answer: $f_c = 2 \times 10^{-4}$ Hz, $C = 160$ mF)

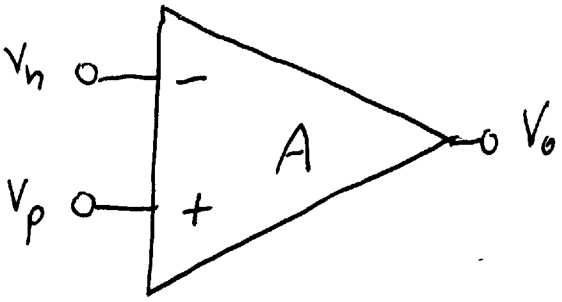
CHAPTER 4

OPAMP CIRCUITS

The operational amplifier is a very versatile component that can be used in many ways,

ranging

- high gain amplifiers
- active filters
- signal generators
- comparators
- Schmitt triggers
- tension followers
- Analog computer:
  integrators / differentiators / exp/log
- current controlled voltage sources.



$V_o = A\,(V_p - V_n)$

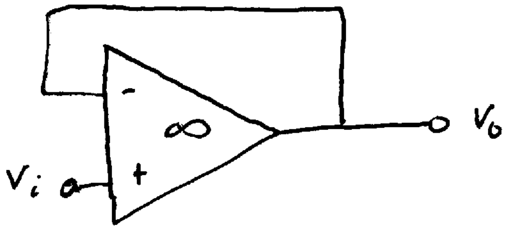
An <u>ideal</u> opamp has

1) infinite gain , $A = \infty$
2) infinite input resistance $r_i = \infty$
3) zero output resistance $r_o = 0$

1a) Because $A = \infty$, $V_p - V_n = V_o / A = 0$

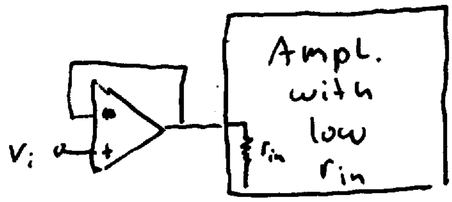
$\Rightarrow \quad V_p = V_n$!

The easiest circuit we can make with an op-amp is a **tension follower**:

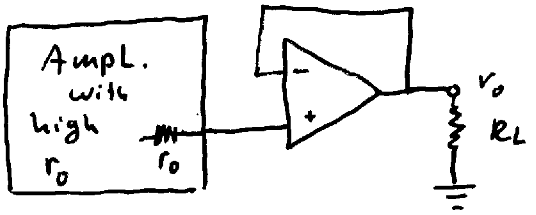


with 100% feedback ($\beta = 1$) and assuming an infinite open-loop gain ($A = \infty$) it is easy to show that $v_o = v_i$.

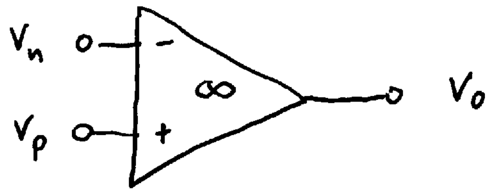
The advantage of this circuit is that it has very high input resistance and low output resistance. This is usefull when we cannot load a high-ohmic source or when we need to drive a low-ohmic input stage.



or



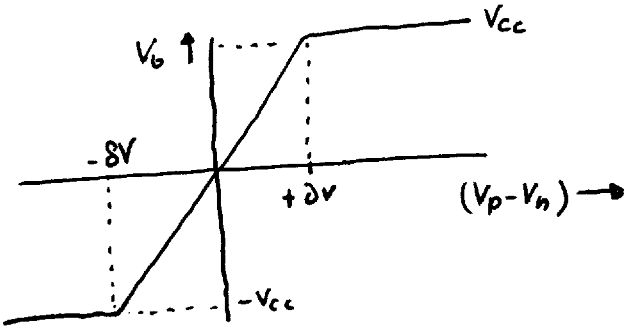
A **comparator** is a circuit that has either V_{cc} or $-V_{cc}$ (the supply voltage) as output, depending on the difference in the input terminals.



$V_o = +V_{cc}$ if $V_p > V_n$

$V_o = -V_{cc}$ if $V_p < V_n$

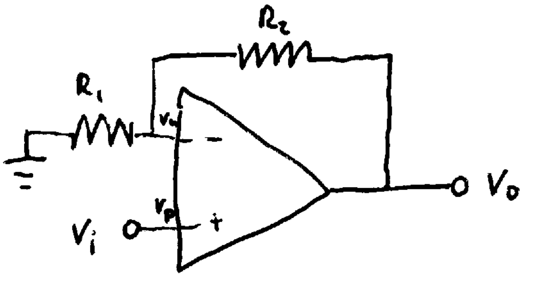
For op-amps with non-infinite gain A, there is a region of input voltages that don't give $\pm V_{cc}$ at the output



It is easy to show that $\partial V = V_{cc}/A$ and is of the order of $10\,V/10^5 \sim 100\,\mu V$ for practical opamps.

A widely used comparator op amp is the "311" which has a high switching speed $-V_{cc} \longleftrightarrow +V_{cc}$

A $\boxed{\text{non-inverting amplifier}}$ can be made by introducing (negative) feedback to the amplifier.



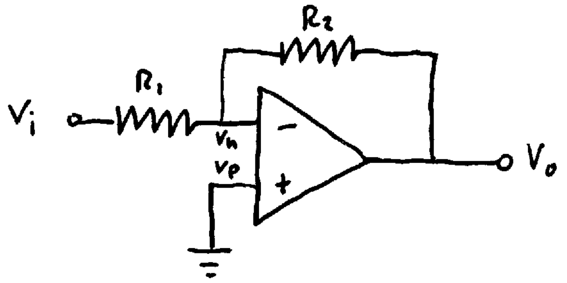
To calculate the gain:

$V_n = V_p = V_i$ (rule 1a of p. 1)

$V_n = \frac{R_1}{R_1 + R_2} V_o$

$$\Longrightarrow \quad \frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1}$$

An **inverting amplifier** is equal to the one above, but with the signal connected to R_1 :



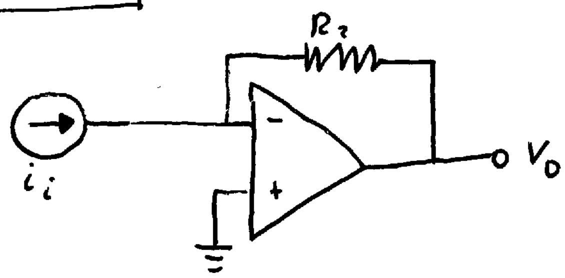
$V_n = V_p = 0\ V$ and that is why the negative terminal in this scheme is ofter called <u>virtual</u> <u>ground</u>.

A current of $I_i = \frac{V_i}{R_1}$ is drawn from the source and (since $r_{in} = \infty$), this current <u>must</u> go through R_2.

The output voltage thus becomes

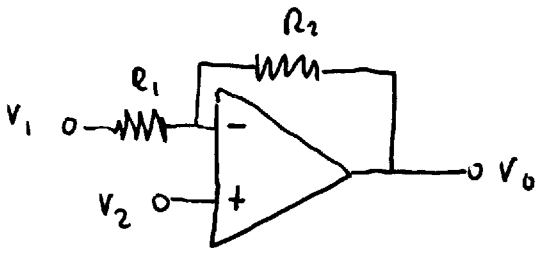
$$V_o = \frac{R_2}{R_1} \cdot V_i \implies \frac{V_o}{V_i} = \frac{R_2}{R_1}$$

The same idea is also used in a **current-to-voltage converter**



$$\frac{V_o}{i_i} = R_2$$

For a **differential amplifier** we connect both inputs :



using superposition we can easily see that

$$V_0 = \frac{R_2}{R_1} V_2 - \frac{R_1 + R_2}{R_1} V_1 \approx \frac{R_2}{R_1} (V_2 - V_1) \qquad (\text{for } R_1 << R_2)$$

we have to always bear in mind that a high gain for $R_2 >> R_1$ (β is very small) will limit the bandwidth. see p.7 of chapter 3:

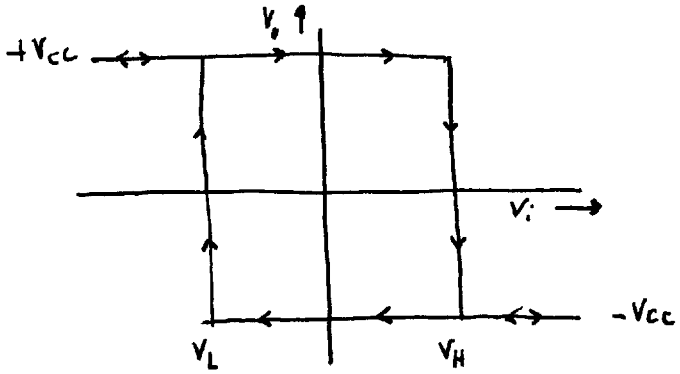
$$f_c = f_0 (1 + A_0 \beta)$$

with f_0 the open-loop bandwidth and A_0 the open-loop gain. In fact, the gain-bandwidth product.

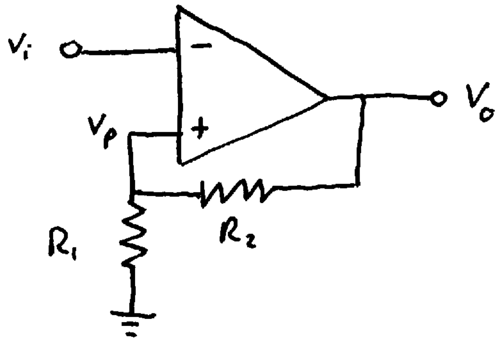
$$GBW = \frac{V_0}{V_i} \times f_c$$

is constant. $\left(\frac{V_o}{V_i} = \frac{A}{1 + A\beta},\ f_c \approx f_0(1 + A\beta) \right),\ GBW = A f_0$

A $\boxed{\text{SCHMITT TRIGGER}}$ is like a comparator, but it has a memory. the output of the schmitt trigger not only depends on the input voltage, but also on the history.



To achieve, positive feedback is used



$$V_p = \frac{R_1}{R_1+R_2} V_o$$

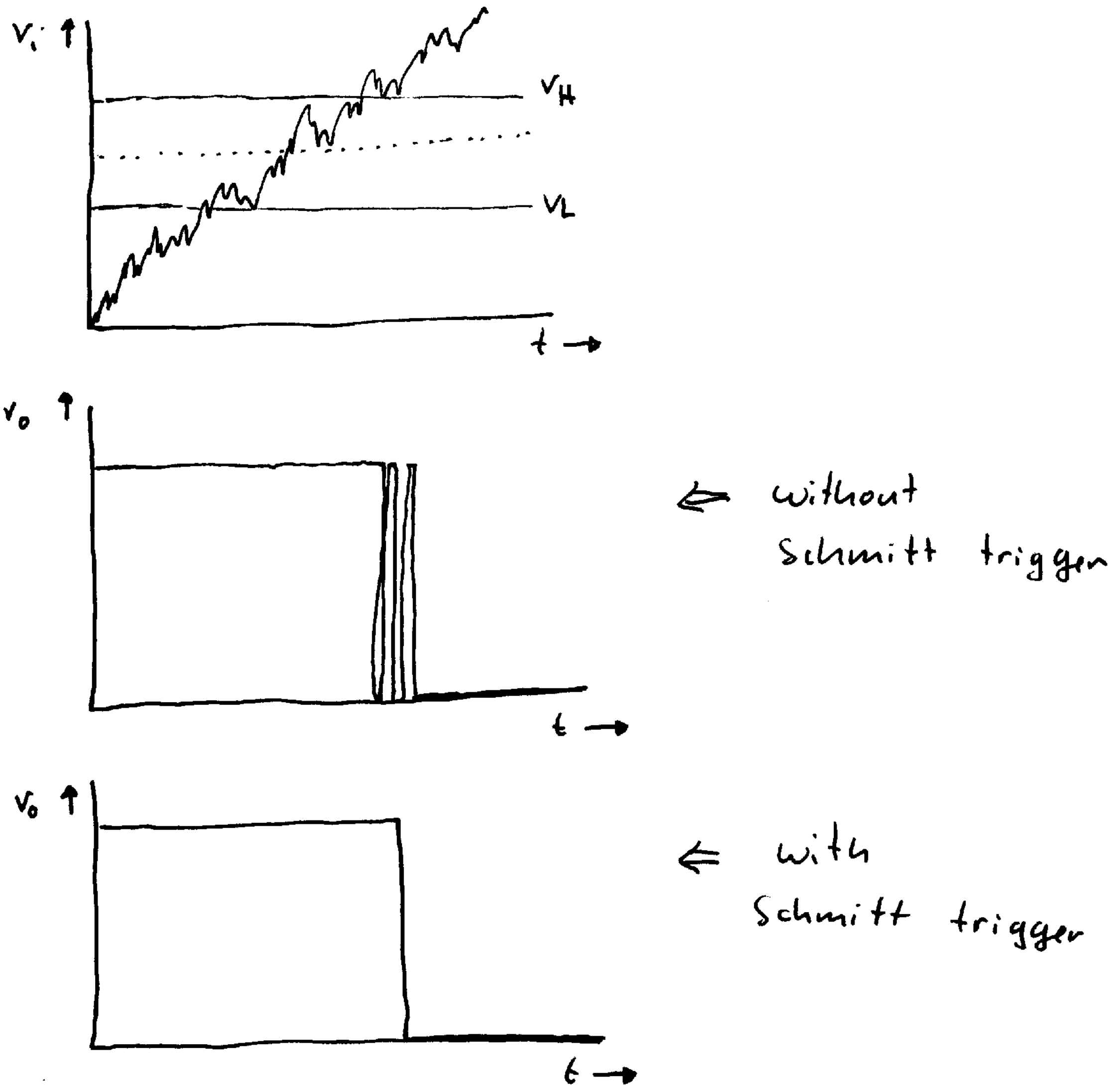
If $V_o = +V_{cc}$, then $V_p = + \frac{R_1}{R_1+R_2} V_o$. Now, if we increase V_i from $-\infty$ upwards, there comes a point where $V_i > V_p$ and thus, since the circuit works as a comparator, V_o becomes $-V_{cc}$. This makes $V_p = - \frac{R_1}{R_1+R_2} V_o$. To make the output commute again, we have to lower V_i to this value.

Thus

$$V_L = - \frac{R_1}{R_1+R_2} V_{cc}$$

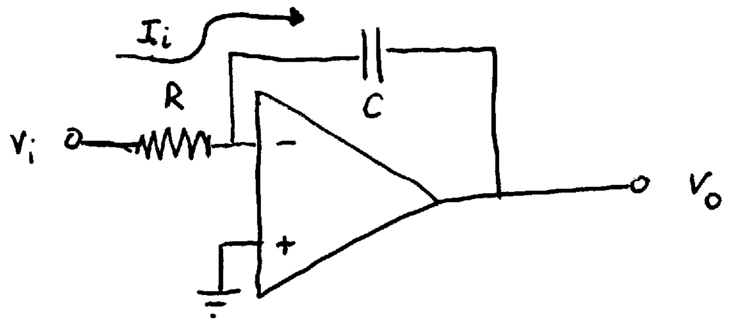
$$V_H = + \frac{R_1}{R_1+R_2} V_{cc}$$

The advantage of a Schmitt trigger lies in the fact of eliminating noise.



Imagine switching on the lights in a room based on a detector . . . .

Integrators and Differentiators are made by putting a condensator in the feedback loop.



⇐ Integrator

The resistor translates the voltage to a current $I_i = V_i/R$. This current cannot enter the opamp and thus is used to charge the capacitor C. The amount of charge in C (assuming at t=0 discharged, Q(t=0) = 0) is equal to the integrated current:

$$Q(t) = \int_0^t I_i(t)\,dt = \int_0^t \frac{V_i(t)}{R}dt$$

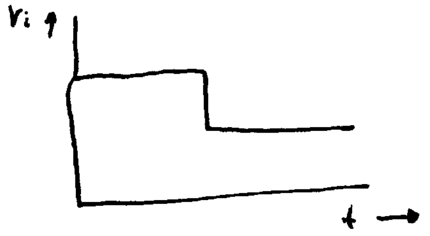
The voltage drop induced by this charge is

$$\Delta V_c = Q/C$$

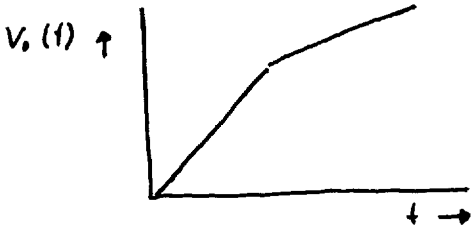
Since one side of the capacitor is connected to virtual ground, the voltage drop is equal to $-V_0$.

Thus

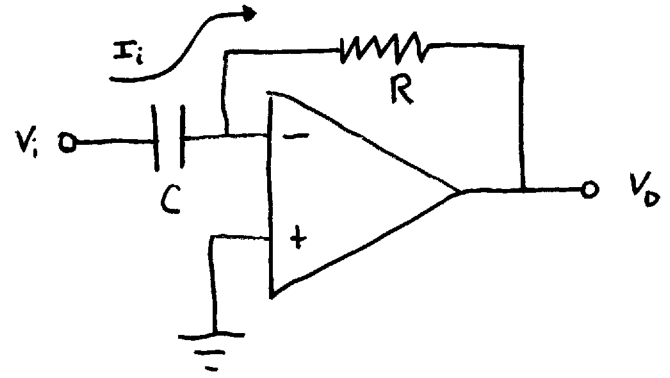
$$V_o(t) = -\frac{Q(t)}{C} = -\frac{1}{CR}\cdot\int_0^t V_i(t)\,dt$$



⟹



To make a **differentiator**, we exchange the resistance and capacitor



The input current I_i is equal to

$$I_i = C \cdot \frac{dv_i}{dt}$$

This current is translated to voltage by R (note that one side of the resistor is at virtual ground and the input resistance of the op-amp is infinite; all current goes through R).

$$V_o(t) = - CR \frac{dv_i(t)}{dt}$$

In both cases, the differentiator and integrator the signals at the output might be limited by the power supply. V_o cannot be larger that $\pm V_{cc}$. In the case of the integrator it means that, for instance, DC signals at v_i can only be integrated up to a certain time. For the differentiator it means that signals cannot change too fast.

with the integrator and differentiator circuits we can build $\boxed{\text{analog computers}}$. For instance for solving differencial equations.

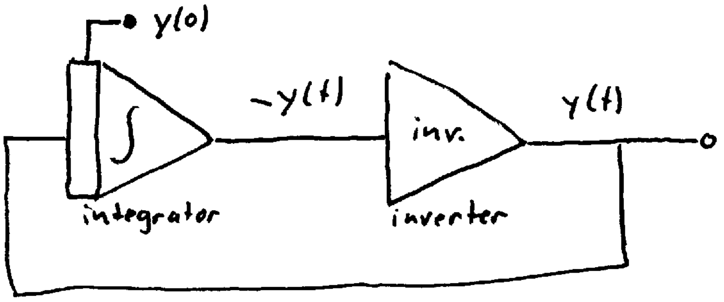
Take for example the differential equation

$$\dot{y} = y \qquad \text{with} \quad y(0) = 1 \qquad \left(\dot{y} = \frac{dy}{dt} \right)$$

This is equivalent to (integrating on both sides)

$$y(t) = y(0) + \int_0^t y(\tau)\, d\tau$$

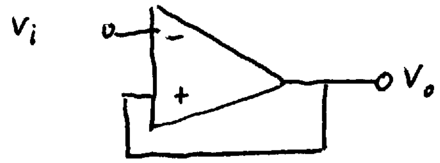
with our op-amp circuits this becomes



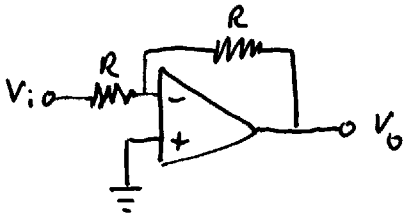
Note 0: The integrator also inverts the signal!

Note 1: the ┌• y(0) part is to load the inverter with the starting value (implies charging the capacitance instantaneously).

Note 2: The inverter can also easily be made of opamps. Why is it not the left circuit?



wrong



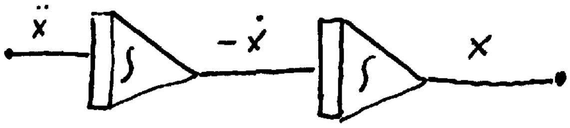
correct

As another example:

$$\ddot{x} + 0.5\dot{x} + x = 4.0 \qquad x(0) = 0, \quad \dot{x}(x) = 1$$

$$\dot{x} = \frac{dx}{dt}, \quad \ddot{x} = \frac{d^2x}{dt^2}$$

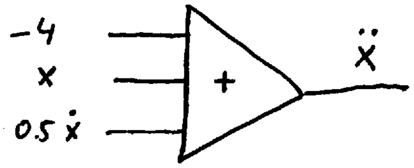
1) Starting with assuming $\ddot{x}$ exists:



2) Rearrange the equation

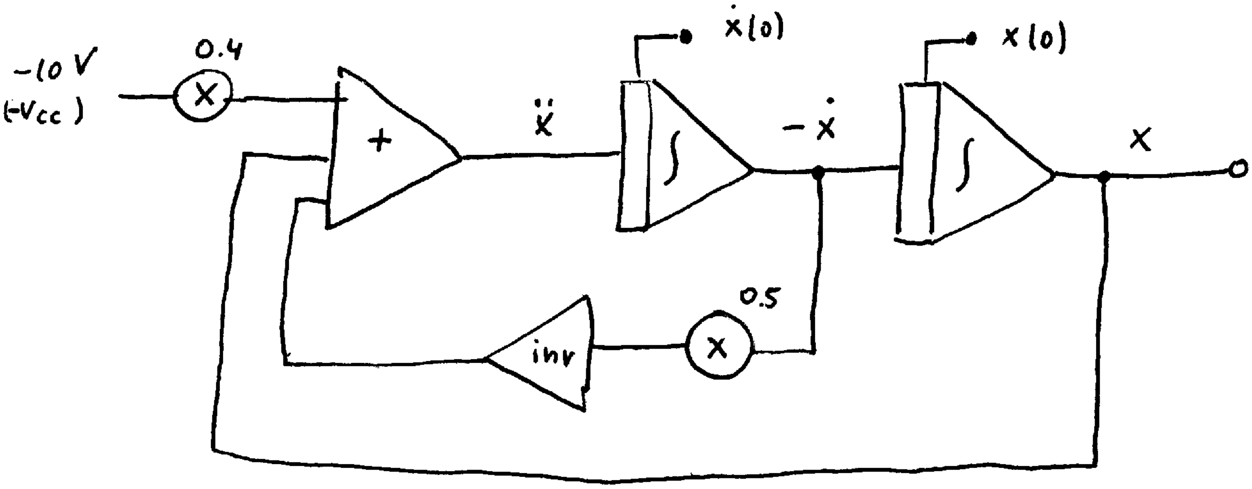
$$\ddot{x} = -0.5\dot{x} - x + 4.0$$

This is equal to

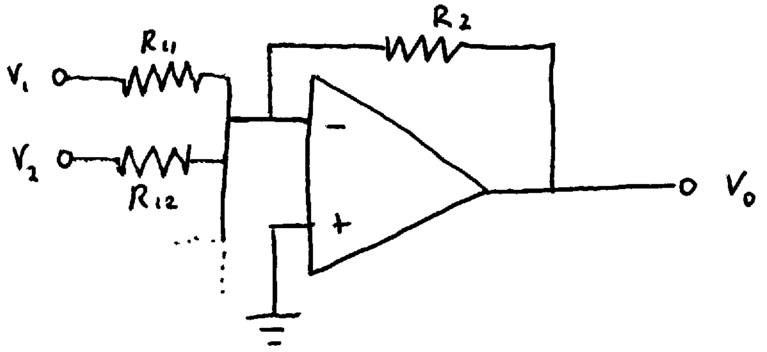


(The summing opamp also inverts!)

3) Now connecting the two parts ("procedures")



- The multipliers $\otimes$ are easily made with opamps. when it is a constant multiplication factor.
- The adder  can also be made with opamps.

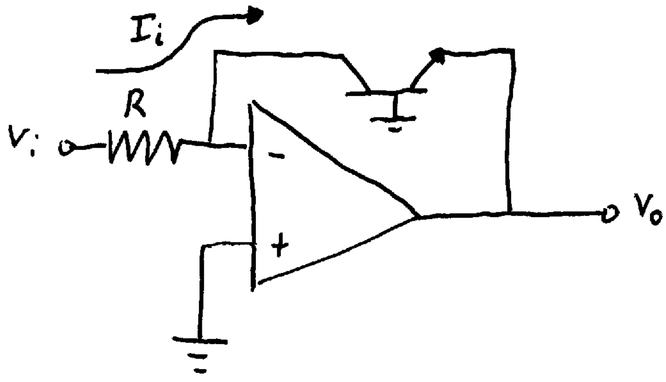


$$V_0 = -\left(\frac{R_2}{R_{11}} V_1 + \frac{R_2}{R_{12}} V_2 + \dots\right)$$

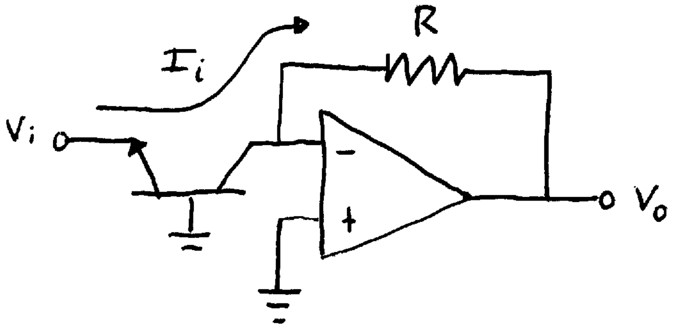
- The circuit on the previous page can be simplified because there are too many inverters, but this is a small detail.

The circuit on the previous page is an example of how a damped oscillation can be calculated using analog electronics. An example of a damped oscillation is a guitar String or a bell. Thus, the above circuit generater musical signals. The first electronic synthesizers were made in this way. We can imagine that the values at $\Gamma^{\bullet}\,\dot{x}(0)$ and $\Gamma^{\bullet}\,x(0)$ were put there by a touch of a key, where the note (frequency) is also determined by the integration constants (RC) of the integrators.

Other "mathematical" circuits are the exponential and logaritmic amplifiers



logarithmic amplifier



exponential amplifier

In the logarithmic amplifier: the input current $I_i = V_i/R$ passes through the transistor. Since

virtual ground

$V_C = 0$ and $V_B = 0$, we can use the diode equation of Ebers - Moll.

$$I_c = V_i/R = I_0 \left\{ \exp\left(\frac{V_{BE}}{V_T} \right) - 1 \right\}$$

Ignoring the -1 and using $V_0 = -V_{BE}$ we see that

$$V_0 = -V_T \ln \left(\frac{V_i}{I_0 R} \right)$$

Note that V_0 has to be positive.

For the exponential amplifier it is not difficult to show that

$$I_i \approx - I_0 \exp\left(-\frac{V_i}{V_T}\right)$$

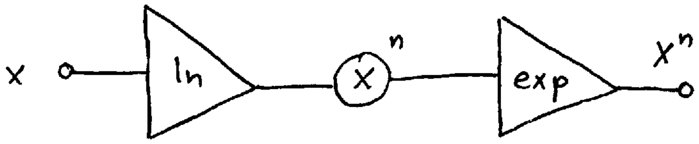
$$V_0 = -I_i R = I_0 R \exp\left(-\frac{V_i}{V_T}\right)$$

(note : V_i has to be negative)

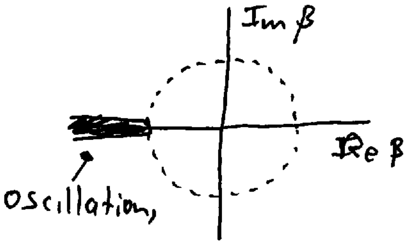
Combining logarithmic and exponential (anti-logarithmic) amplifiers we can make calculations of the type

$$y = x^n$$

$$y = \exp(n * \ln(x))$$

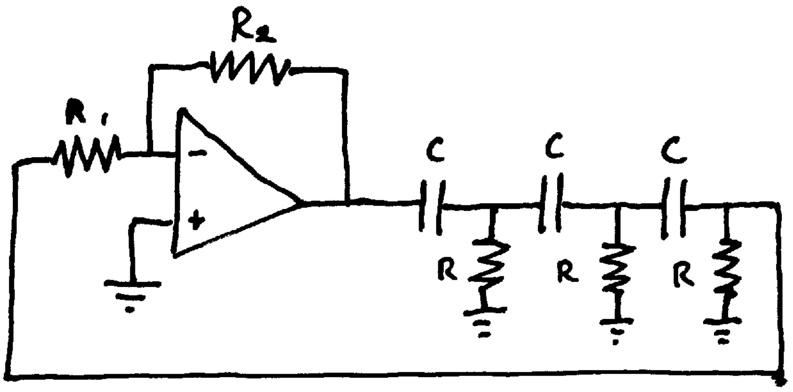


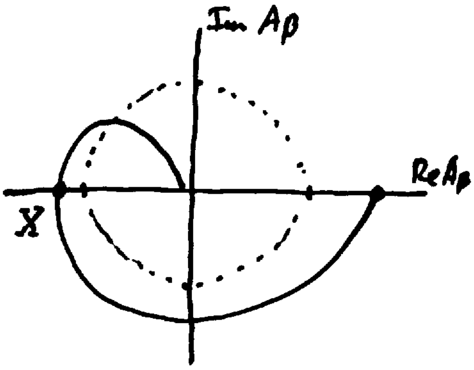
Oscillators. In chapter 3 we saw how to avoid oscillations. Now we will use the same analysis to make oscillations happen. We need $A\beta$ to become real and <-1 for some frequency (see Nyquist ← plot here).



Assuming a flat A, without poles and phase changes, we can see that 3 poles are needed in β to achieve that.

For example



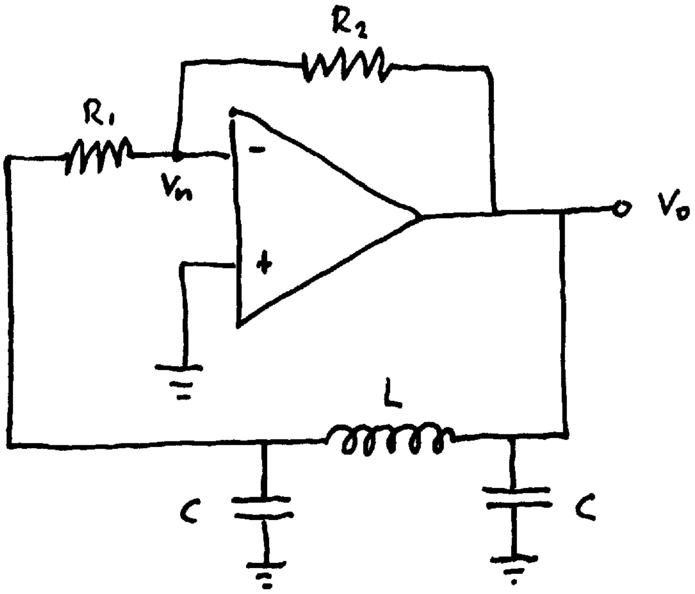


$$\beta = \frac{R^3}{(R^3 - 5R / \omega C) + j\,((1/\omega c)^3 - 6R^2/\omega c)}$$

In the oscillation frequency, at X, the imaginary part of β is zero.

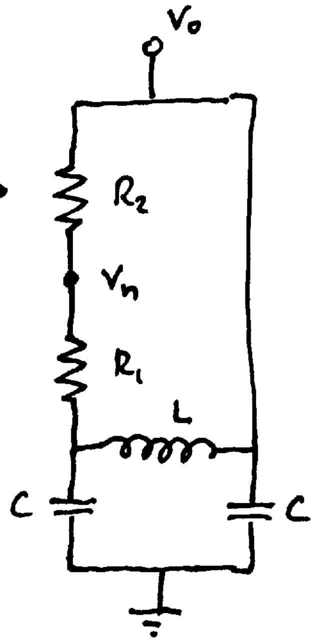
$$\left(\frac{1}{\omega c}\right)^2 = 6R^2 \Rightarrow \omega = \frac{1}{\sqrt{6}} RC$$

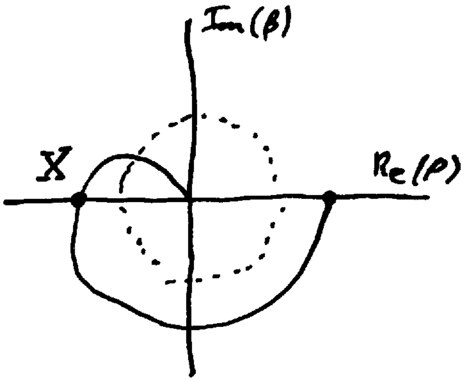
Another example is the Colpitts Oscillator. It consists of an opamp with negative feedback, so it needs also three poles. In this case they are made by two capacitances and one coil.



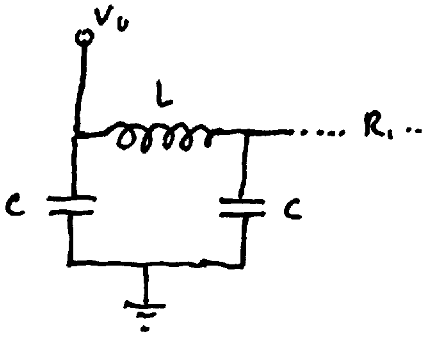
Colpitts Oscillator

The feedback β loop can be visualized as → It is not easy to calculate $\beta = \frac{V_n}{V_o}$, but we can make a simplification:





At X the feedback must be real (imaginary part = 0). Plus, if we assume R_1 to be large, it must mean that the impedance from the output of the amplifier looking into the CLC bridge must be real:



$$Z = \frac{1}{j\omega C} \parallel \left(j\omega L + \frac{1}{j\omega C} \right)$$

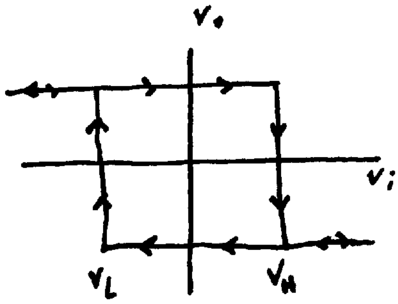
$$= \frac{\frac{1}{j\omega C}\left(j\omega L + \frac{1}{j\omega C} \right)}{\frac{1}{j\omega C} + j\omega L + \frac{1}{j\omega C}}$$

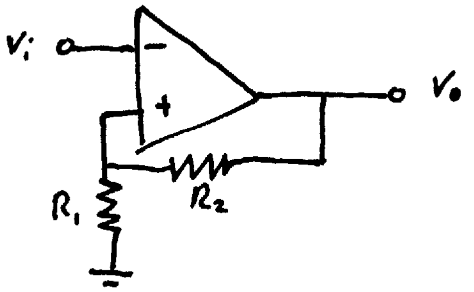
$$Z = \frac{(L/C - 1/\omega^2 C^2)}{j\left(-\frac{1}{\omega C} + \omega L - \frac{1}{\omega C}\right)} \quad \text{must be real}$$

$$\Rightarrow \quad \frac{-2}{\omega C} + \omega L = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2}{LC}} \,, \quad f = \frac{\omega}{2\pi} = \sqrt{\frac{1}{2\pi^2 LC}}$$

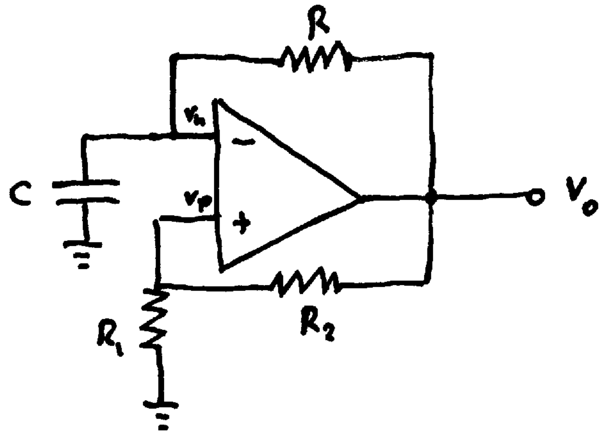
A $\boxed{\text{memory element}}$ can be made of a Schmitt trigger

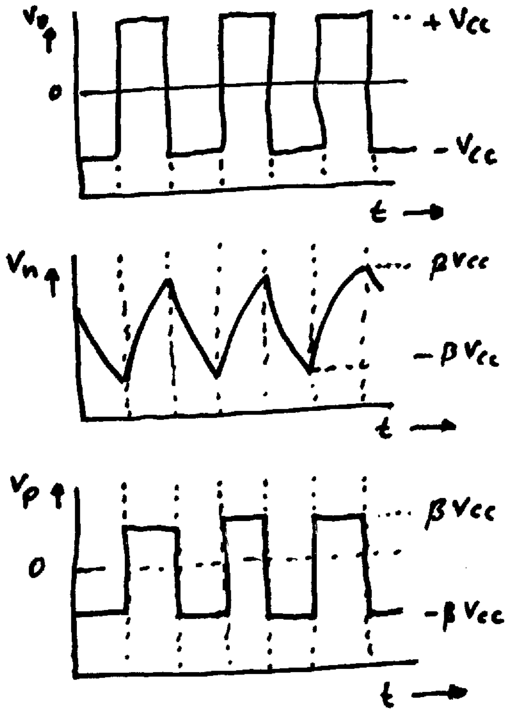




For $V_i = 0$, there are two possible states at the output $+V_{cc}$ and $-V_{cc}$, depending on the history. Thus, for $V_i = 0$, we can read the memory element. To program the element, a voltage higher than $V_H = \frac{R_1}{R_1 + R_2} V_{cc}$ should be set at V_i. This makes the output to $-V_{cc}$ (logical "0"). To program a logical "1", a voltage below $V_L = -\frac{R_1}{R_1 + R_2} V_{cc}$ should be set at the input.

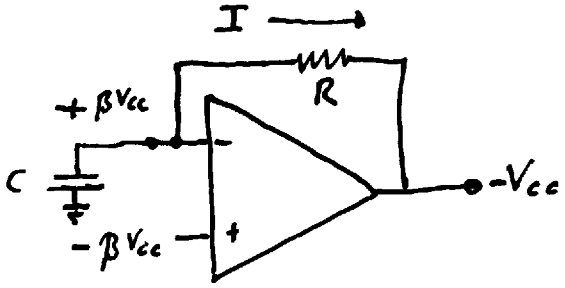
A memory element is a bistable element. It can take either of two output values and is stable. Contrasting this is a $\boxed{\text{Astable Multivibrator}}$ which is astable at both possible output values ± V_{cc}





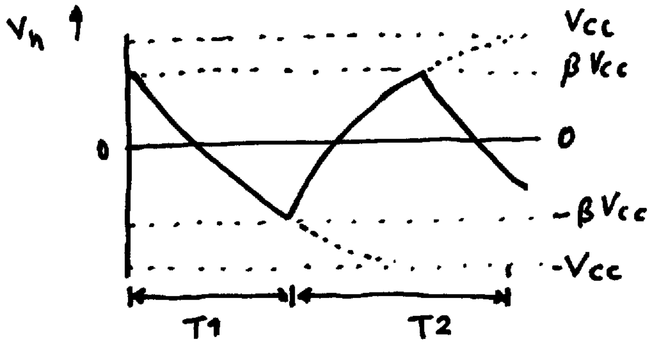
$$\beta = \frac{R_1}{R_1 + R_2}$$

Imagine V_o at $-V_{cc}$. V_p is then at $-\frac{R_1}{R_1+R_2} \cdot V_{cc}$ $= -\beta V_{cc}$. Imagine V_n starts at $+\beta V_{cc}$.



Through the resistance R will initially $(t=0)$ go a current $I = (\beta V_{cc} - (-V_{cc}))/R$.

This current will uncharge C! Because of this, V_n is lowering (Note that thus I is constantly reduced and is not constant) It is thus exponentially decaying from $+\beta V_{cc}$ to $-V_{cc}$, with an RC time of $R \times C$. However, when it reaches $-\beta V_{cc}$ it becomes lower than V_p. In this case the output commutes $-V_{cc} \rightarrow +V_{cc}$ and a reverse cycle starts.



How long does it take to charge the capacitor?

1) Starts at $+\beta V_{cc}$
2) Aiming at $-V_{cc}$

$$\left.\begin{array}{l}\\ \\ \end{array}\right\} V_n = -V_{cc} + (\beta+1) V_{cc} \exp(-t/\tau)$$

3) The relaxation time is $\tau = RC$ $\rightarrow$ $V_n = -V_{cc} + (\beta+1) V_{cc} \exp(-t/RC)$

4) Stops at $V_n = -\beta V_{cc}$

$$\Longrightarrow \quad T_1 = RC \ln\left(\frac{1+\beta}{1-\beta}\right)$$

In a similar way we can show that $T2 = T1$ and in total

$$T = 2\,RC \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$f_{osc} = \frac{1}{2\pi T}$$

CHAPTER 5

OUTPUT STAGES

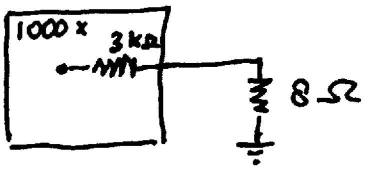
cH 9 Sedra
cH 16 Bogart

Up to now we have analyzed the circuits without worrying about what was connected to it. In fact, we have always assumed the ideal case, with nothing connected.

Moreover, the signals at the various stages of the amplification were alway small signals, thus guaranteeing that it was working in the linear regime. In this way, the only frequency appearing at the output was the one supplied at the input.

When the output signal is not small, non-linearities might occur. This can be put in the so called total - harmonic distortion (THD), which describes the percentage of power appearing at the harmonics $2 * f_o$, $3 * f_o$ ..., etc of the fundamental f_o.

Another important parameter is the output resistance r_{out} of the total circuit. Our signal amplifiers discussed in the previous chapters (differential pair and common emitter amplifier) all had high output resistance ($\sim R_c, \sim k\Omega$'s). When connecting a load to this high-ohmic amplifiers, the gain will drop significantly. Consider the following amplifier to which we connect speakers ($\sim 8\,\Omega$)



The gain drops from 1000 to $1000 \cdot \frac{8}{8+3000} = 2.7$ (a factor 376 !).

We need an output stage with low resistance at the output. From lectures in circuit analysis, we know that, given an amplifier with R output resistance, the maximum power transfer is when the load resistance is R.

Although the reasoning cannot be inversed (!), we will use the expected load resistance as an indication of the desired output resistance.

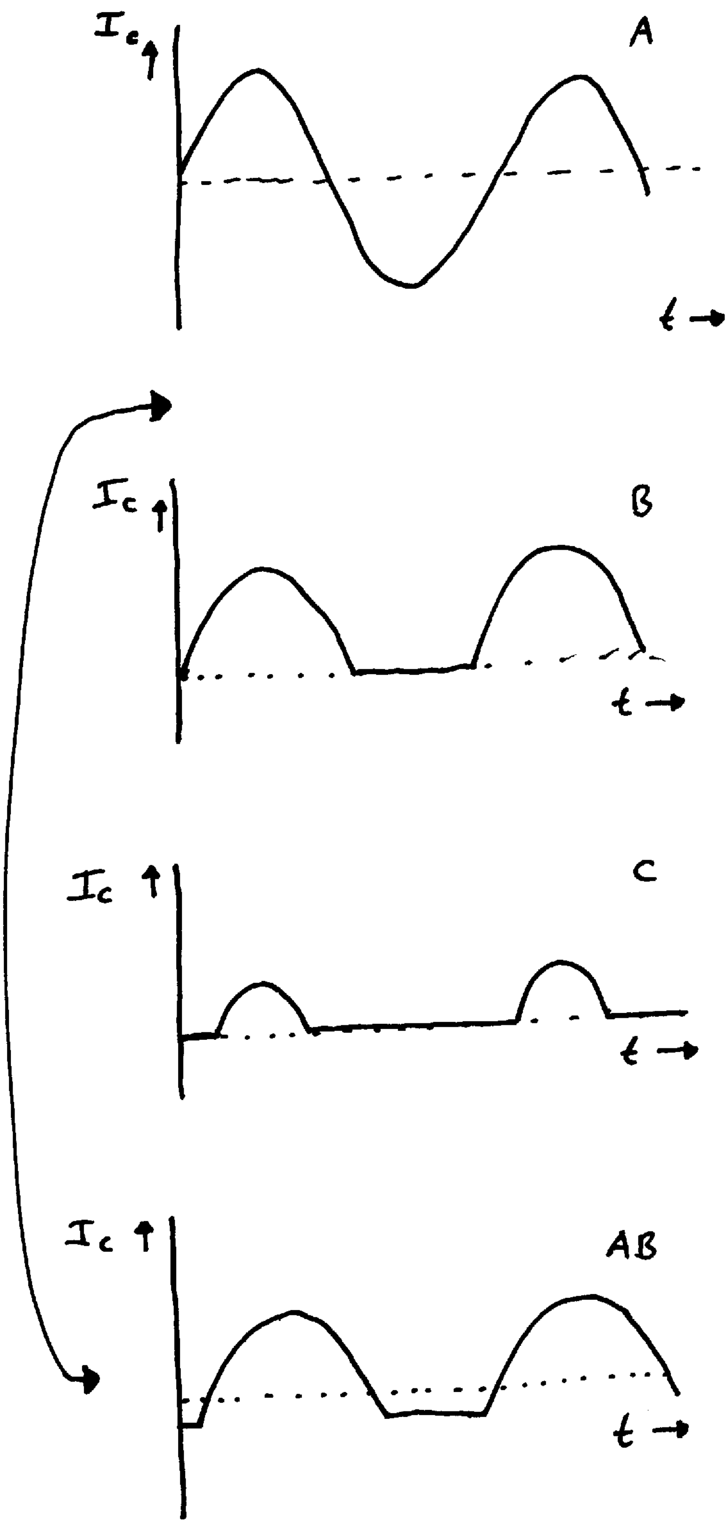
Another important aspect is the heat generated inside the components. Ideally, the heat is only generated outside the amplifier, in the load. However, significant heat can be generated in the transistors. Such transistors can overheat.

Finally, it is important to know what the output signals can be. Are they limited by any voltage (for example: only possive voltages). We will start with this.

Classification of output stages

The classification of amplifiers is based on the waveform of the collector current at the output when a sinusoidal input signal is applied.

In this way can be distinguished the various "classes" of amplifiers, see the figures on the next page.



"class A"

360° of sine appears
at output
100%

"class B"

180° of sine appears
at output. 50%

"class C"

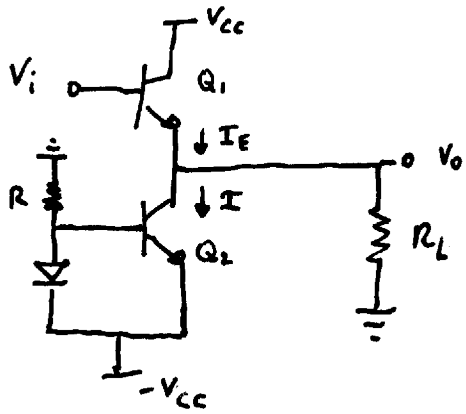
< 180° of sine appears
at output. < 50°

"class AB"

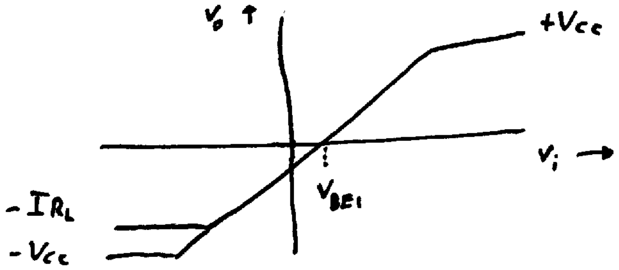
between 180° and 360°
appears at output
>50%, < 100%

Examples of the various classes :

CLASS A :



The bottom part is a current source defining the current I. (see p. 13 of ch. 1)



The gain is 1. The output voltage is limited by the powe supply, either cutting at +Vcc by Q_1 or -Vcc by Q_2. Moreover, I_E has to be positive. Thus V_o has to be bigger than $-I \cdot R_L$.

<u>power conversion efficiency</u>

$$\eta \equiv \frac{P_L}{P_s} \begin{array}{l} \leftarrow \text{power at load} \\ \leftarrow \text{power supplied to output stage} \end{array}$$

The maximum swing at v_o is from $-V_{cc}$ to $+V_{cc}$

The average power P_L is thus

$$P_L = \int_0^T v(t) \cdot I(t) \cdot dt = \int_0^T V_{cc} \sin(\omega t) \cdot \frac{V_{cc}}{R_L} \sin(\omega t)$$

(T is full period)

$$= \frac{1}{2} \frac{V_{cc}^2}{R_L}$$

The $-V_{cc}$ power supply supplies a constant current of $2I$ (Q_2 + diode). The current supplied by $+V_{cc}$

swings from 0 to $2I$. The average is thus I.

$$P_S = \frac{1}{T}\int_0^T (-V_{cc})\cdot(-2I)\,dt + \frac{1}{T}\int_0^T (V_{cc}\cdot I + V_{cc}\cdot I\cdot \sin \omega t)\,dt$$

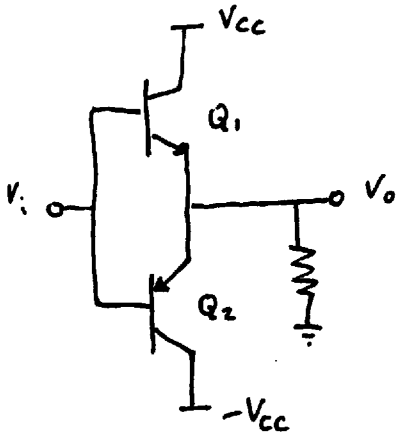
$$= 3\,V_{cc}\,I$$

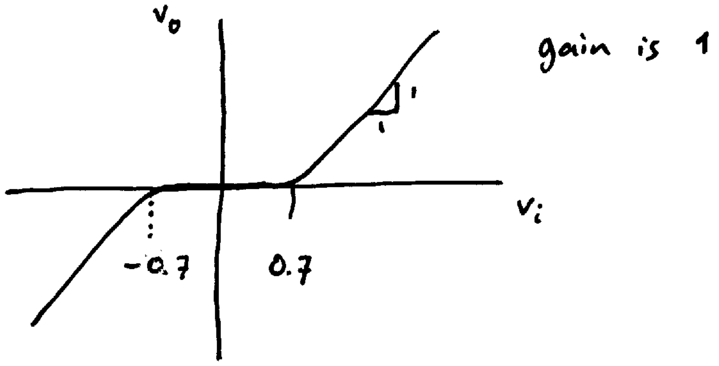
$$\eta = \frac{P_L}{P_s} = \frac{1/2\; V_{cc}^2/R_L}{3\,V_{cc}\,I} = \frac{1}{6}\,\frac{V_{cc}}{I R_L}$$

For a good polarization $I = V_{cc}/R_L$. Thus, the maximum efficiency is 16%.

CLASS B:

CLASS A = 2 × CLASS B





This class B output stage consists of an npn and a pnp transistor in series. They cannot both conduct at the same time, thus eliminating the quiescent power consumption. The price paid is an inactive region $-0.7\ V \ldots +0.7\ V$.

maximum efficiency : (ignoring the 2×0.7 V losses)

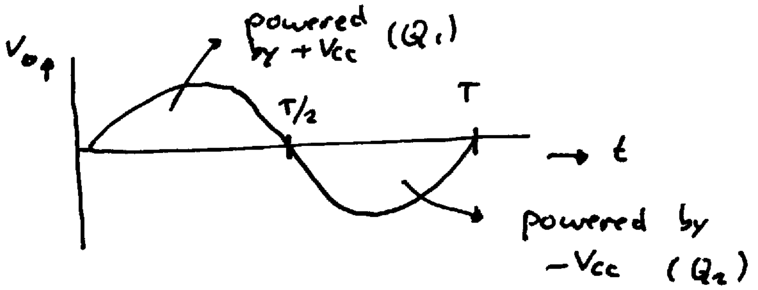
$$\eta = \frac{P_L}{P_S}$$

P_L is the same as the other circuit

$$P_L^{max} = \frac{1}{2} V_{cc}^2 / R_L = \frac{1}{T}\int_0^T v_o(t) \cdot v_o(t) / R_L \, dt$$

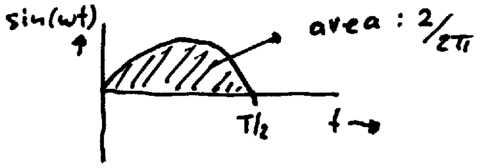
P_S is different . $P_S = \frac{1}{T}\int_0^T V_{cc} \cdot I_{cc} \, dt$

$$P_S^{max} = \frac{1}{T}\int_0^{T/2} V_{cc} \cdot \frac{v_o(t)}{R_L} \, dt + \frac{1}{T}\int_{T/2}^{T} -V_{cc} \cdot \frac{v_o(t)}{R_L} \, dt$$



max : $v_o(t) = V_{cc} \sin(\omega t)$

$$P_S = \frac{2}{T}\int_0^{T/2} \left(V_{cc}^2 / R_L\right) \sin(\omega t) \, dt$$

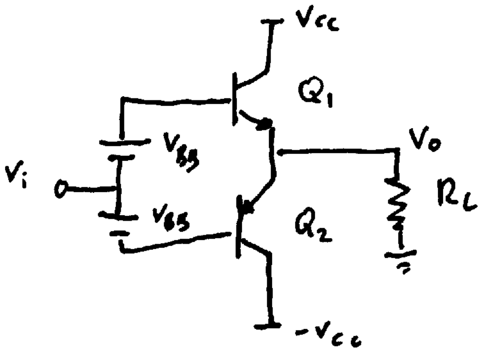


$$= \frac{4}{2\pi} V_{cc}^2 / R_L$$

$$\eta = \frac{P_L}{P_S} = \frac{1/2 \, V_{cc}^2 / R_L}{4/2\pi \, V_{cc}^2 / R_L} = \frac{\pi}{4} = 79 \%$$

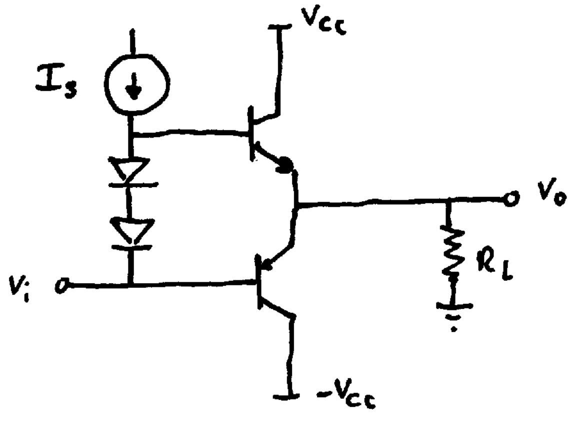
a significant increase compared to the class A output stage of p. 6

To avoid the cross-over distortion from -0.7 V to +0.7 V, the transistors can be biased. Instead of 2 x B, the result is 2 x AB:



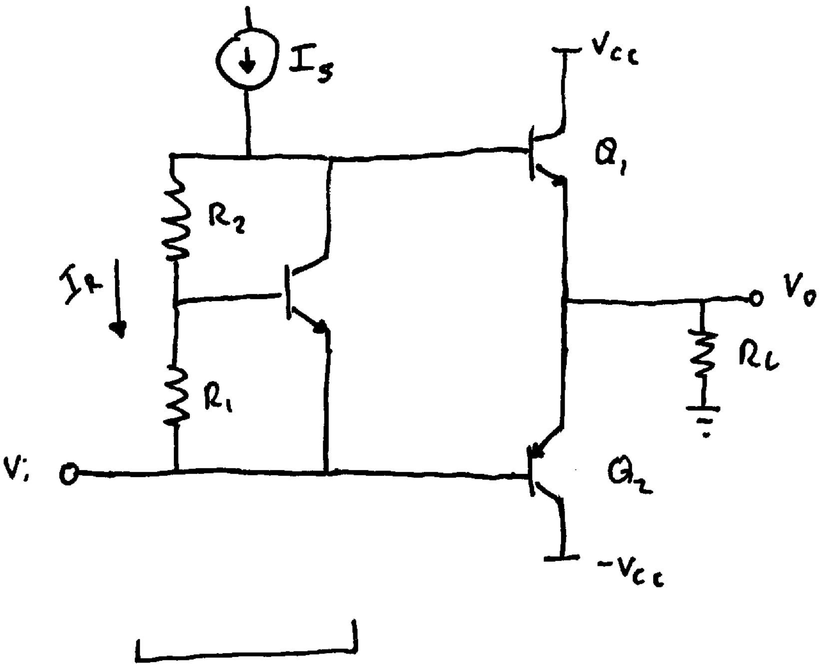
V_{BB} should be chosen in this way to marginally open the two transistors at $v_i = 0$. Too much bias will cause an unnecessary quiescent current ($I_{E1} \neq 0$, $I_{E2} \neq 0$ for $v_i = 0$). Too little bias will cause "cross-over" or "dead band".

An elegant way to implement this is with a current source and diodes:



The diodes guarantee that both transistors are marginally open at $v_i = 0$. (Note that $V_o = 0.7$ V for $v_i = 0$ V)

Another way to do it is by an V_{BE} multiplier (see next page)



V_{BE} multiplier

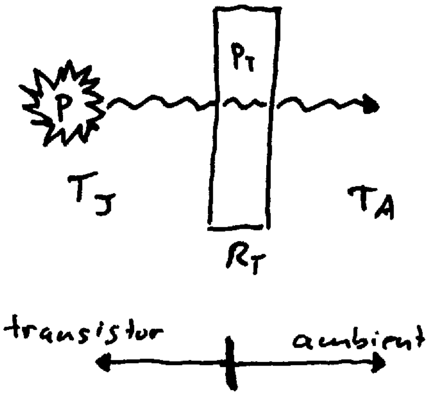
Ignoring the base current of the V_{BE} multiplier, it can be shown that the current I_R causing a voltage drop of V_{BE} (0.7 V) in R_1 must cause a voltage drop of $R_2 \cdot I_R = R_2 \cdot \frac{V_{BE}}{R_1}$ in resistance R_2. The total drop in the V_{BE} multiplier is thus

$$V_{BE} + \frac{R_2}{R_1} V_{BE} = V_{BE} \left(1 + R_2/R_1\right)$$

Heat generated by transistors

A Transistor cannot exceed a temperature of 150°C – 200°C. With higher temperatures they get distroyed irreversibly

The power generated inside a transistor causes a rise in temperature of the junction. This heat is transported to the ambient



T_A : ambient temperature
T_J : junction temperature
P : Power generated at junction
P_T : Power transported to ambient
R_T : Thermal resistance of package.

The transport of heat to the ambient, P_T, is limited by the thermal conductivity of the materials (package, air, etc.). This can be modelled by the parameter R_T, the thermal resistivity. The heat transported is proportional to the temp. gradient:

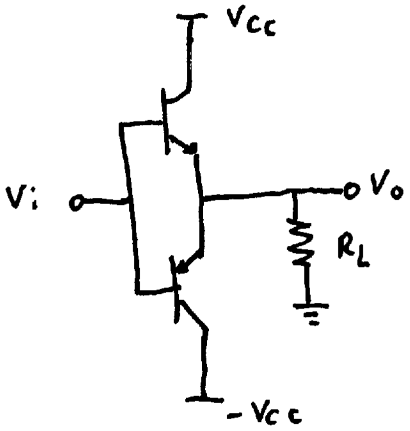
$$P_T = (T_J - T_A)/R_T$$

When equilibrium is established, the heat transported is equal to the heat generated, P. It is easy to show that the final temperature is then

$$T_J = T_A + P \cdot R_T$$

(Note: unit of R_T is K/W or °C/W)

Example



$R_L = 8\,\Omega$, $R_T = 80\,K/W$, $T_A = 25\,^\circ C$

we want 25 W output power

$P_L^{max} = 25\ W$

Since $P_L^{max} = \frac{1}{2} V_{cc}^2 / R_L$, we

design $V_{cc} = 20\ V$

Then

$$P_L = \frac{1}{2} V_{cc}^2 / R_L = \frac{1}{2}\ 20^2/8 = 25\,W$$

$$P_S = \frac{2}{\pi} V_{cc}^2 / R_L = 31.8\ W$$

power dissipated inside the transistors

$$P = P_S - P_L = 6.8\ W$$

Per transistor

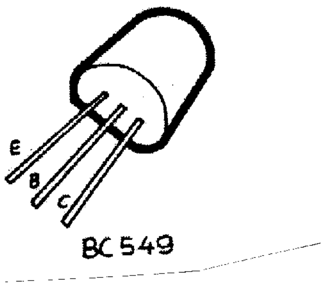
$$P = 3.4\ W$$

Then, the junction temperature will be

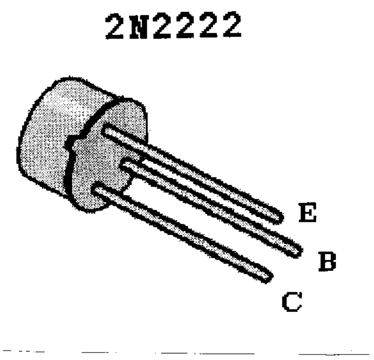
$$T_J = 25\,^\circ C + 3.4\ W \times 80\,^\circ C/W = 297\ ^\circ C$$

This transistor will burn! A way to prevent this is to use special metal transistors where the collector (which generates most of the heat) is in direct contact with the metal case and use heat sinks. or use ventilators on top of the transistors

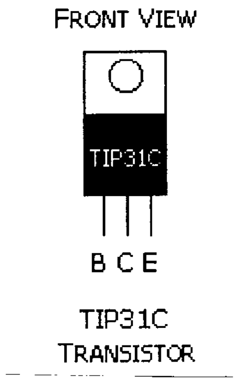
Decreasing R_T



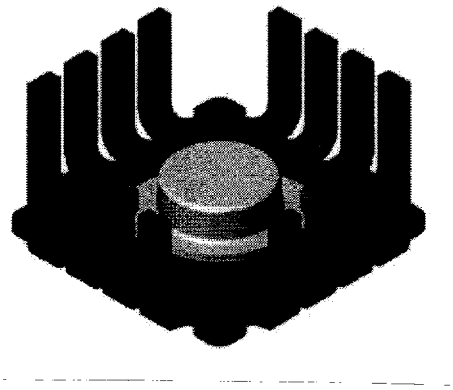
← Simple transistor with plastic packing ex. BC 549



← transistor with metal casing ex. 2N 2222



← transistor with heat sink to screw to circuit board



← transistor with metal case and extended heat sink

Below is a schematic of a "741" operational amplifier. Identify the input stage (differential pair of ch. 1) and output stage of this chapter. (copied from Sedra, p. 812)

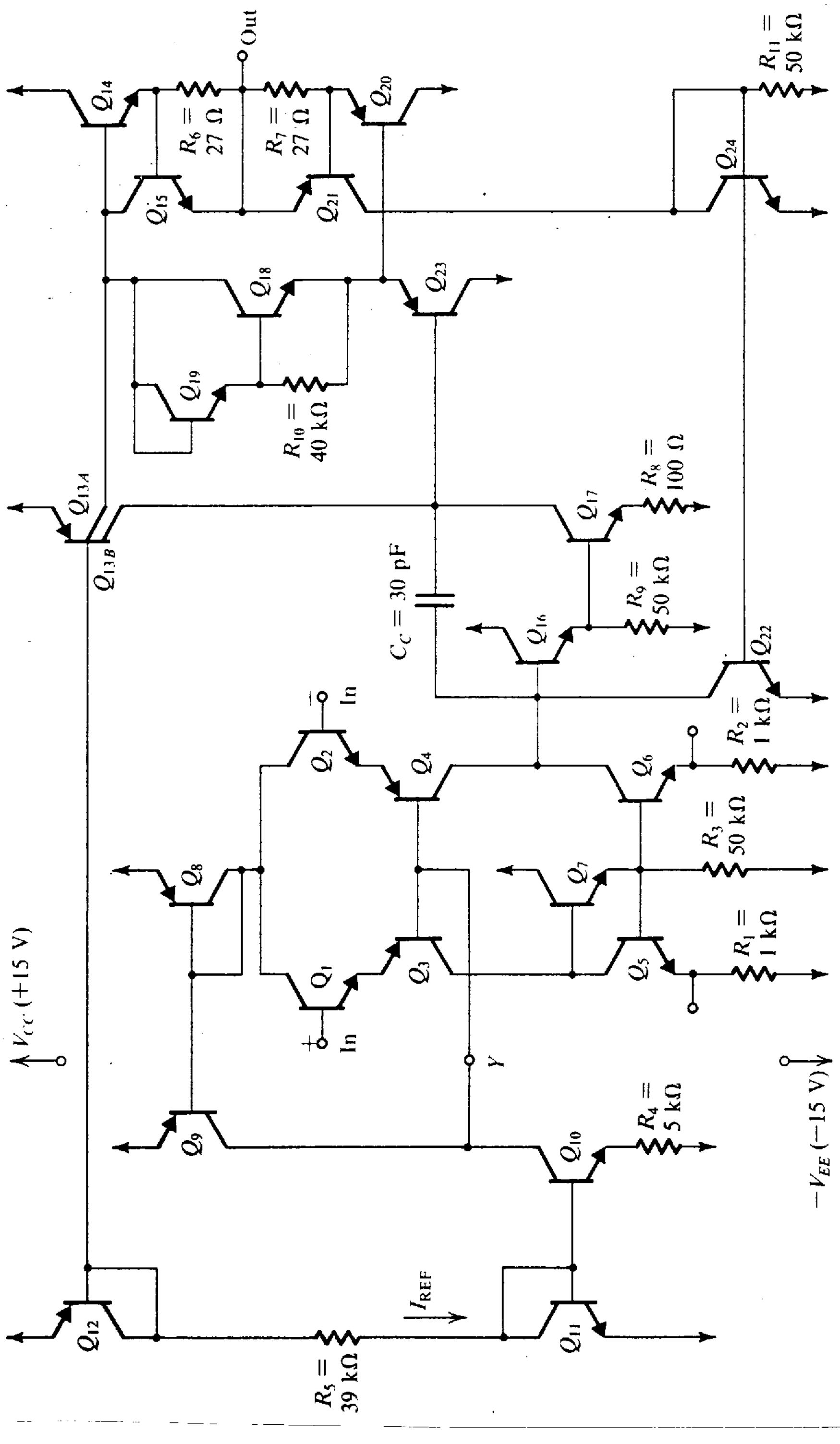


Fig. 10.1 The 741 op-amp circuit. Q_{11}, Q_{12}, and R_5 generate a reference bias current, I_{REF}. Q_{10}, Q_9, and Q_8 bias the input stage, which is composed of Q_1 to Q_7. The second gain stage is composed of Q_{16} and Q_{17} with Q_{13B} acting as active load. The class AB output stage is formed by Q_{14} and Q_{20} with biasing devices Q_{13A}, Q_{18} and Q_{19}, and an input buffer Q_{23}. Transistors Q_{15}, Q_{21}, Q_{24}, and Q_{22} serve to protect the amplifier against output short circuits and are normally off.