CHAPTER 2

FREQUENCY RESPONSE

Ch. 7 Sedra

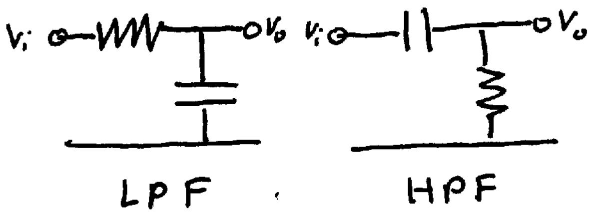
One of the most important parameters of electronic circuits is the frequency response. Normally we want as high a bandwidth as possible, but not always. Sometimes we want to cut off the DC part (for decoupling or to reduce power consumption or noise). Sometimes we want to limit the high-frequencies (to avoid oscillations). Sometimes both (bandpass amplifier).

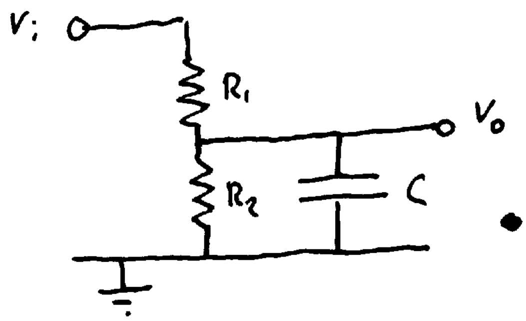
For advanced circuits (with many capacitors and inductors) the calculation is very complicated. The best thing is then to use electronics simulators such as Electronics Workbench an P-Spice.

However, it is very usefull to get an idea of the frequency response. In order to do that we will simplify our analysis. We will look for individual cut-off frequencies and then combine these in the final frequency response. Thus, the strategy becomes very simple:

> Find filters like the LP and HP filters presented in Chapter 0

As an example :

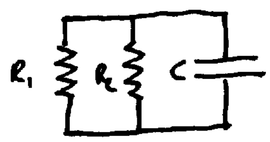




- This filter looks like a low-pass filter

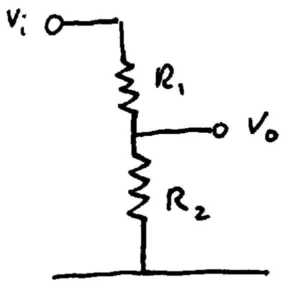
- Because it has only 1 condensator, it has only one cut-off frequency
- To find this frequency we have to find the effective resistance this C sees. For this, we connect the input to ground and we get this circuit.

$R_{eff} = R_1 // R_2 \Rightarrow \omega_0 = C\,(R_1 // R_2)$



$$\Rightarrow \quad T(s) = \frac{A_{DC}}{1 + s \Big/ \frac{1}{(R_1 // R_2)C}}$$

- The DC gain A_{DC} can be found by considering the condensator as open-circuit:



$$A_{DC} = \frac{R_2}{R_1 + R_2}$$

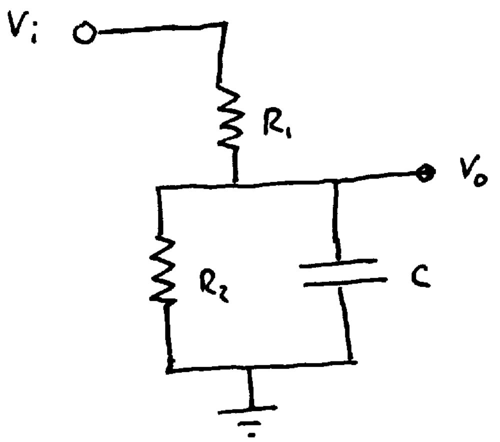
Thus

$$T(s) = \frac{R_2 / (R_1 + R_2)}{1 + s / \frac{1}{(R_1 // R_2)\, C}}$$

with $R_1 // R_2 = R_1 R_2 // (R_1 + R_2)$ this becomes

$$T(s) = \frac{1/R_1 C}{s + 1/(R_1 // R_2)\, C} \qquad (*)$$

The method we learned at Circuit Analysis results (of course) in the same:



This is a voltage divider with the impedance of a condensator as $\frac{1}{sC}$:

$$\frac{V_o}{V_i} = \frac{\left(\frac{1}{sC} // R_2\right)}{\left(\frac{1}{sC} // R_2\right) + R_1}$$

After some rearranging of terms the same (*) expression emerges. Try it.

Which method do you prefer ?

Bode plots

As a simplification of our analysis we will look for simple filters in our circuit: Either LPF's or HPF's. In other terms, we will look for single-pole (LPF) or single-zero (HPF) filters, so-called first-orde functions. On basis of this we will draw a Bode plot observing the following rules:

- we start with a horizontal line ($A = A_0$)
- Every pole (LPF) introduces a cut-off frequency <u>above</u> which the slope is changed by -20 dB/dec.
- Every zero (HPF) introduces a cut off frequency <u>below</u> which the slope is changed by +20 dB/dec

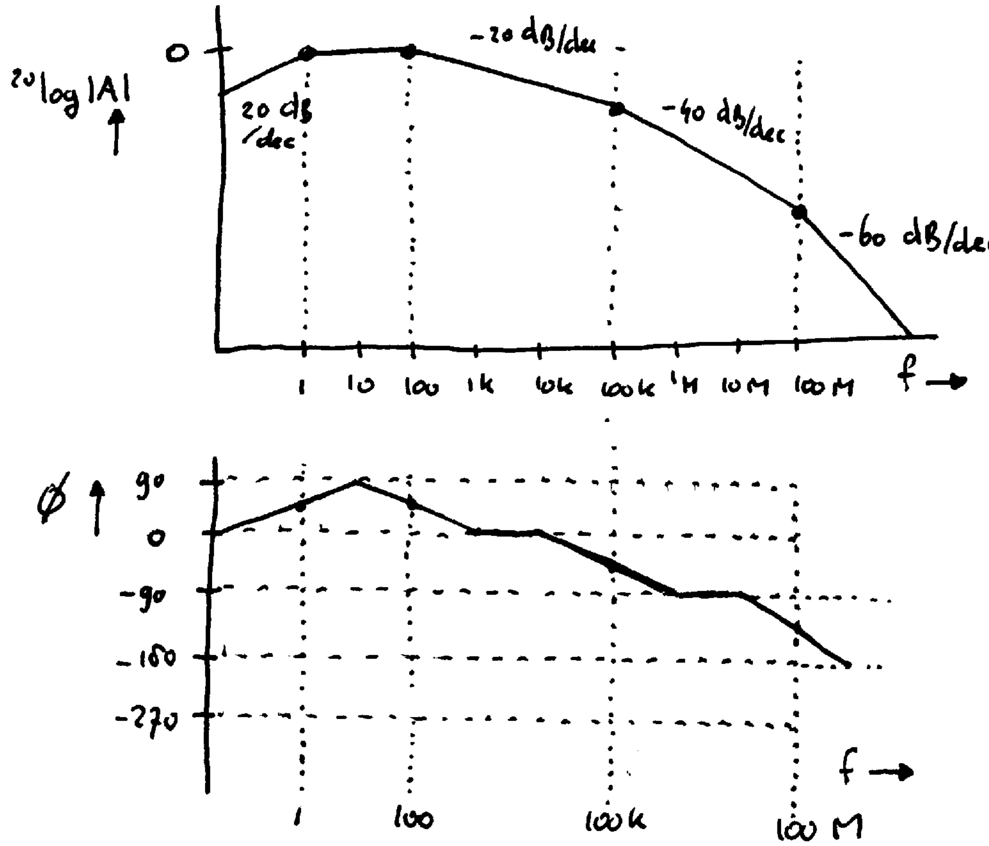
At the phase plot:

- we start with $0°$
- The phase changes $-90°$ in approximately 2 decades (factor 100 in frequency) around the cut-off frequency introduced by every pole (LPF). $-45°$ before f_c, $-45°$ after f_c. At f_c: $\Delta\phi = -45°$
- The phase changes $+90°$ in approximately 2 decades around the cut-off frequency introduced by every zero (HPF). $+45°$ before f_c, $+45°$ after f_c. At f_c: $\Delta\phi = +45°$

Example :

what is the Bode plot of a filter (passive, $A_{max} = 1$) with three poles (100 Hz, 100 kHz, 100 MHz) and one zero (1 Hz) ?

The mid band gain is 1 (with 90° shift).



Note that these are approximations

The real functions are smoother

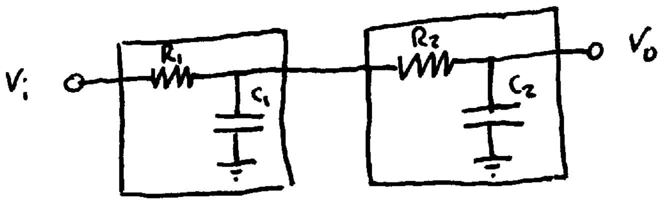
$$T(f) = \frac{1}{(1 + jf/100)} \cdot \frac{1}{(1 + jf/10^5)} \cdot \frac{1}{(1 + jf/10^8)} \cdot \frac{1}{(1 + 1/jf)}$$

f =		1		100		100k		100 M	
$\lvert T(f)\rvert$ =	$\sim f$		~ 1		$\sim 1/f$		$\sim 1/f^2$		$\sim 1/f^3$
ϕ =	$0°$	$+45°$		$+45°$		$-45°$		$-135°$	
		$\frac{1}{\sqrt{2}}$	——	$\frac{1}{\sqrt{2}}$					

Bandwidth of filter : $100 - 1 = 99$ Hz

This is a band-pass filter.

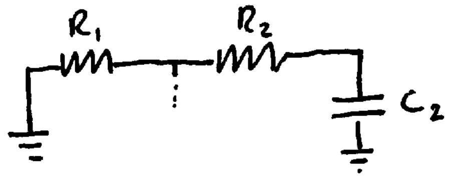
Warning 1 : Don't be misled



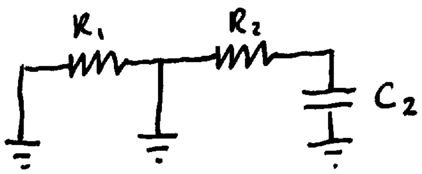
What are the two poles of this filter ?

simple thought : $\omega_1 = \frac{1}{R_1 C_1}$, $\omega_2 = \frac{1}{R_2 C_2}$. wrong !

For instance : we have to find the effective R that C_2 sees, while connecting V_i to ground. This might be $R_1 + R_2$ (if C_1 is considered open-circuit) or just R_2 (if C_1 is considered short-circuit). Which one is correct depends on values of R_1 R_2 C_1 and C_2.



or

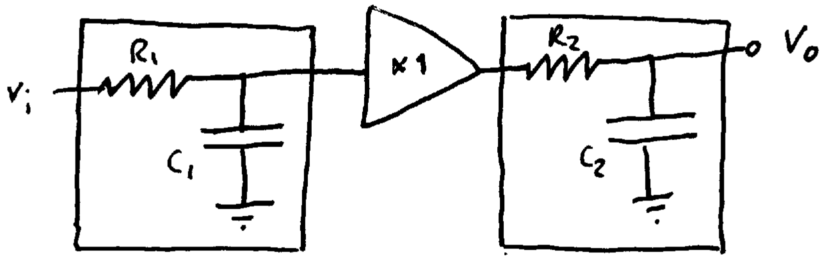


If the pole of filter 1 is much lower than that of the pole we are trying to find for filter 2 then we use the right circuit, because $f_2 >> f_1 \Rightarrow$ The first filter is already blocking $\rightarrow$ C_1 is effectively a short circuit. If the pole of filter 1 is much higher, then we use the left circuit and find $\omega_2 = \frac{1}{(R_1+R_2) C_2}$

There are many ways to go to Rome. Many ways to analyze. In case of doubt $\rightarrow$ Spice

Warning 2 : multi pole/zero

Even if the individual filters are nicely decoupled, as in the figure below, we have to be carefull.



note : ideal amplifier (op amp) has $r_{in} = \infty$, $r_{out} = 0$

Imagine $R_1 = R_2 = R$ and $C_1 = C_2 = C$. Two identical filters. what is the cut-off frequency of the total circuit?

$$\omega_1 = \frac{1}{RC}, \quad \omega_2 = \frac{1}{RC} = \omega_1 \Rightarrow \omega_{tot} = \frac{1}{RC} \; ? \quad \text{wrong!}$$

Remember : at the cut off frequency the amplitude is $\frac{1}{\sqrt{2}}$ of mid band amplitude, Per definition!

At $\omega = \omega_1$: $|T(\omega_1)| = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$, because

$$|T(\omega)| = \frac{1}{\sqrt{1 + \omega^2/\omega_1^2}} \cdot \frac{1}{\sqrt{1 + \omega^2/\omega_2^2}} \Rightarrow \frac{1}{2}$$

Easy calculation shows $(\omega_2 = \omega_1)$:

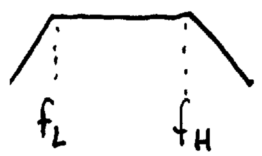
$$|T(\omega_{tot})| = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{1 + \omega_{tot}^2/\omega_1^2} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\omega_{tot} = \sqrt{\sqrt{2} - 1}\; \omega_1 \sim 0.64\; \omega_1$$

So, if, for instance, we link two 100 Hz LPF's together, the total cut-off frequency is 64 Hz.

In the same way: for two equal HPF's $\omega_{tot} = 1.55\; \omega_1$

Normally what is interesting is to find the pass-band. That is to say the lower and upper frequencies of band that has a gain larger than $\frac{1}{\sqrt{2}}$ times the maximum gain. $f_L \dots f_H$.



* when the individual lower cut-off frequencies $f_{L1}, f_{L2} \dots f_{LN}$ are well seperated, or when at least the higher one is well seperated, then

$$f_L \approx \mathrm{MAX}\,(f_{Li})$$

* The same accounts for the higher cut-off frequency f_H (when "well seperated")

$$f_H \approx \mathrm{MIN}\,(f_{Hi})$$

("well seperated" means more-or less a factor 4)

For instance:

$f_{L1} = 1$ Hz, $f_{L2} = 2$ Hz, $f_{L3} = 2.4$ Hz, $f_{L4} = 30$ Hz

$f_{H1} = 200$ kHz, $f_{H2} = 3$ MHz, $f_{H3} = 3.3$ MHz

$\Rightarrow f_L \sim 30$ Hz, $f_H \sim 200$ kHz

* when not well seperated it can easily be calculated as

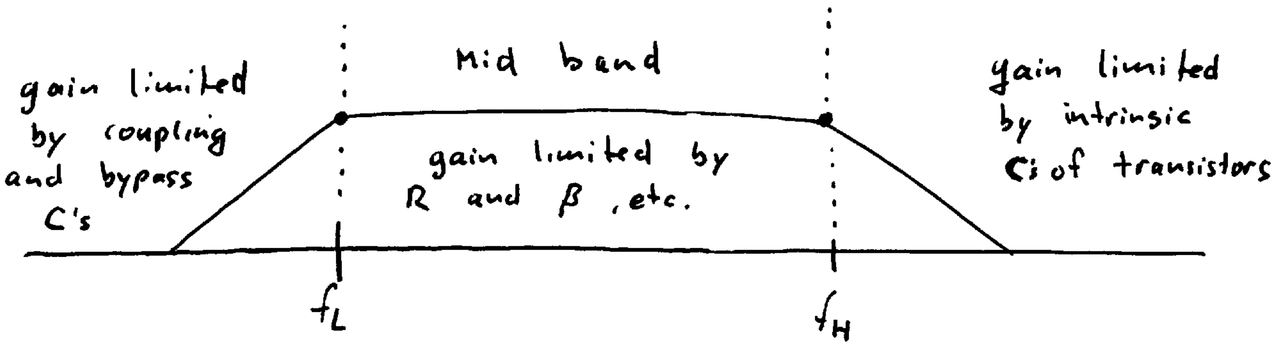
$$f_L \approx \sqrt{f_{L1}^2 + f_{L2}^2 + \dots f_{LN}^2}$$

$$\frac{1}{f_H} \approx \sqrt{\frac{1}{f_{H1}^2} + \frac{1}{f_{H2}^2} + \frac{1}{f_{H3}^2} \cdots \frac{1}{f_{H4}^2}}$$

As can easily be verified these definitions reduce to the previous definitions for well seperated cut-off frequencies.

In every textbook you can find a different approach. They are all approximations. It is more important to understand where the cut-off frequencies are coming from – what is limiting the circuit – then to exactly know these frequencies.

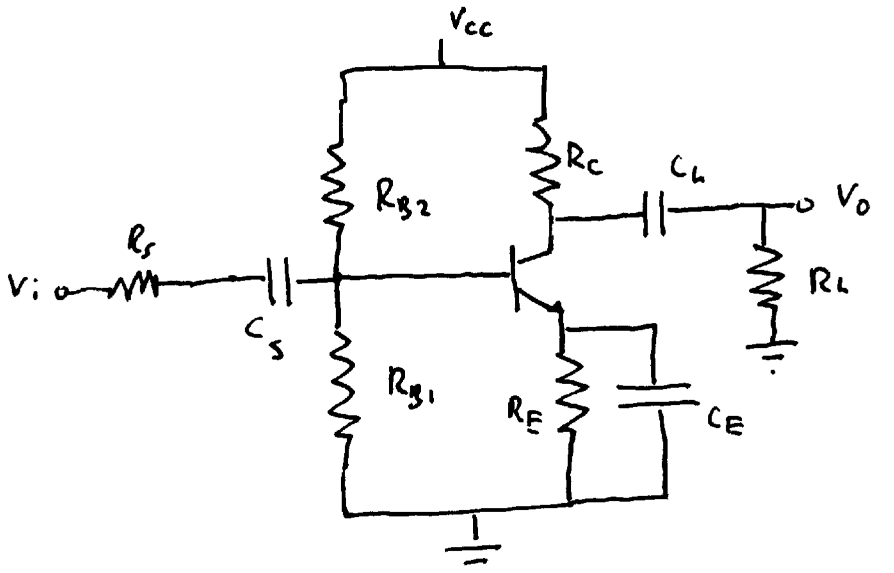
First design the circuit, then simulate it with SPICE. Then build it and test it in practice.



Frequency Response of the common-emitter amplifier :

p 608 Sedra

we will now add (more) coupling condensators and a bypass condensator. to our CEA.
The function of a coupling condensator is to remove the DC part of the input or output signal. The function of a bypass ~~c~~condensator is to increase the midband gain, as we will see, maintaining the bias polarization conditions.



C_L, C_S : coupling capacitors

C_E : bypass capacitor.

The capacitors C_S, C_L and C_E each introduce a lower cutoff frequency ; they function as high-pass filters. For C_S and C_L this is easy to see. Remember that a capacitor is short circuit for high frequencies and open-circuit for low frequencies. Why is C_E also a high-pass filter ?

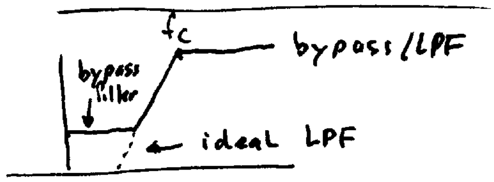
Remember that the gain of a CEA is equal to the resistance at the collector divided by the resistance at the emitter. Thus

$$\boxed{A = \frac{R_c}{R_E + r_e}} \quad \text{low frequencies}$$

and

$$\boxed{A = \frac{R_c}{r_e}} \quad \text{high frequencies}$$

Although this is not really a low-pass filter ($A_{DC} \neq 0$), we can consider it as such.



ideal LPF has a zero at $f = 0$ Hz and a pole at f_c

bypass filter has a zero at $f \neq 0$ Hz and a pole at f_c

For the low frequency behavior we thus have 3 filters. One determined by C_S, one by C_L and one by C_E. Which one is domination. For C_S, should we consider C_L and C_E as open circuit or close circuit? (It will determine what effective resistance C_S sees and what will be the cut-off frequency.) One trick we can use is the method of

SHORT - CIRCUIT TIME CONSTANTS
TO DETERMINE LOWER FREQUENCY

SHORT CIRCUIT TIME CONSTANTS METHOD

⇒ source to ground
⇒ Consider every other capacitor as short circuit (For instance, for C_S consider C_E and C_L as short circuit).

⇒ Determine $\tau_i = C_i R_{eff,i}$ for every capacitor in this way.

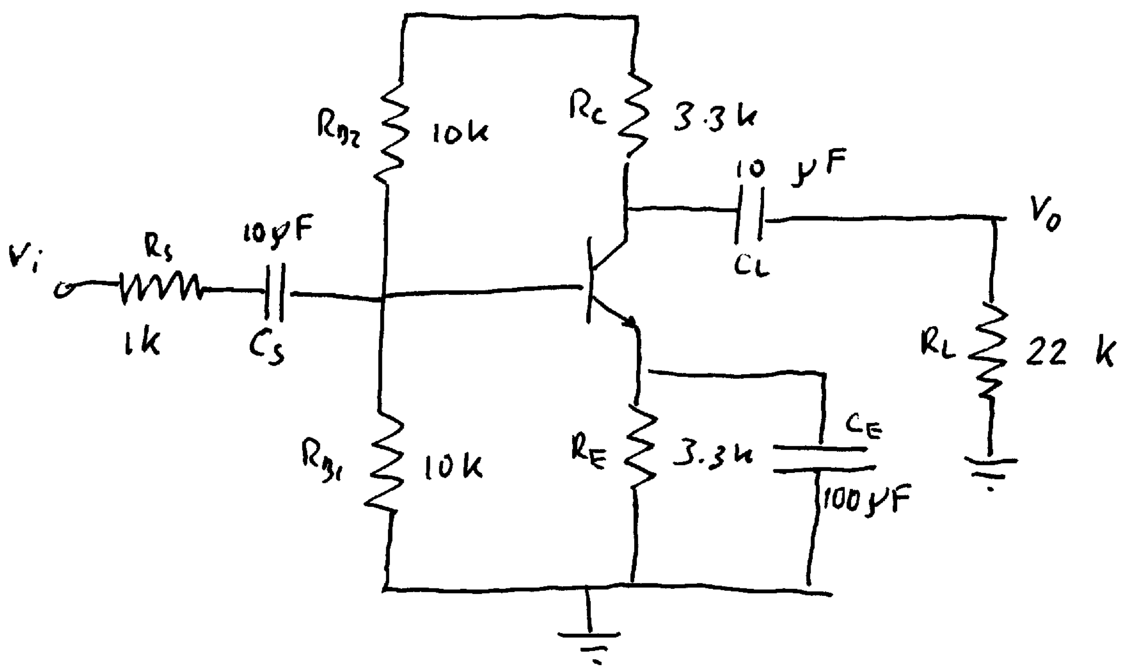
⇒ $$\tau_{tot}^{-1} = \sum \frac{1}{\tau_i}$$

⇒ $$\omega_L = \frac{1}{\tau_{tot}} = \sum_i^N \frac{1}{\tau_i}$$

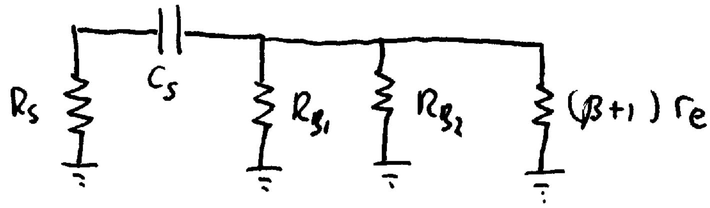
FOR LOW FREQUENCIES

without proof: while this will not give correct results for the individual time constants, the final result for ω_L is correct (p 690 of Sedra).

As a byproduct, we will immediately see which capacitor is the limiting one.



C_S: $\tau_S = 10\,\mu F \cdot R_{eff}$. R_{eff} : short circuit all other C's:

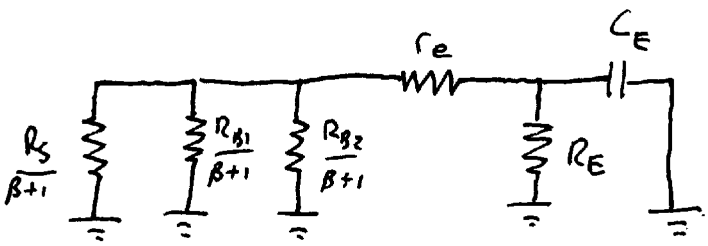


(note : R_E bypassed)

$r_e = 20\ \Omega$
$r_\pi = 2\ k\Omega$

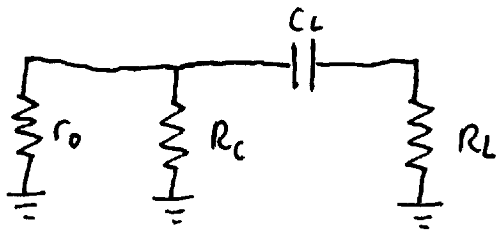
$$R_{eff} = R_S + (R_{B1} // R_{B2} // r_\pi)$$
$$= 1k\Omega + (10\,k\Omega // 10\,k\Omega // 2\,k\Omega)$$
$$= 2.43\ k\Omega$$
$$\tau_S = 10\,\mu F \times 2.43\ k\Omega = 24\ ms \quad \left(f_{LS} = \frac{1}{2\pi\tau} = 6.5\ Hz\right)$$

C_E : $\tau_E = 100\,\mu F \cdot R_{eff}$



$$R_{eff} = R_E // (r_e + (R_{B1} // R_{B2} // R_S)/\beta+1)$$
$$= 3.3\,k\Omega // (20\ \Omega + (10\,k\Omega // 10\,k\Omega // 1\,k\Omega)/101)$$
$$= 3.3\ k\Omega // (20\ \Omega + 8.3\ \Omega)$$
$$= 28\ \Omega$$
$$\tau_E = 100\,\mu F \times 28\ \Omega = 2.81\ ms \quad \left(f_{LE} = \frac{1}{2\pi\tau} = 56.7\ Hz\right)$$

C_L . $\tau_L = 10\,\mu F \times R_{eff}$



$$r_o \approx \frac{V_A}{I_c} \approx 200\ k\Omega$$

$$R_{eff} = (r_o // R_C) + R_L$$

$$= (200\ k\Omega\ //\ 3.3\ k\Omega) + 22\ k\Omega$$

$$= 25.2\ k\Omega$$

$$\tau_L = 10\mu F \cdot 25.2\ k\Omega = 0.25\ s \quad \left(f_{LL} = \frac{1}{2\pi\tau_L} = 0.63\ Hz\right)$$

---

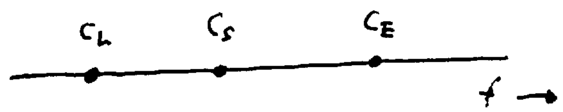
conclusions

$$① \quad \tau_{tot}^{-1} = \sum_{i=1}^{N} \frac{1}{\tau_i} = \underbrace{\frac{1}{2.81\ ms}}_{C_E} + \underbrace{\frac{1}{24\ ms}}_{C_S} + \underbrace{\frac{1}{250\ ms}}_{C_L} \Rightarrow$$

$$\tau_{tot} = 2.5\ ms$$

$$\omega_L = \frac{1}{\tau_{tot}} = 400\ rad/s, \quad f_L = \frac{\omega_L}{2\pi} = 64\ Hz$$

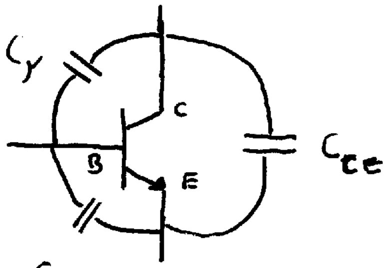
② The bandwidth is limited by the capacitor giving the shortest time constant : C_E.
If we want to increase the bandwidth, we should increase C_E or decrease β (different transistor) or increase R_s.



High - frequency analysis

The high frequency response is normally limited by the intrinsic capacitances of the transistors.
These capacitances are unavoidable and have a physical nature (see my option lectures "Physics of semiconductor

devices"). We can find the values for the capacitances in the data sheets of the transistors used



capacitances of an npn transistor

C_μ is capacitance between collector and base

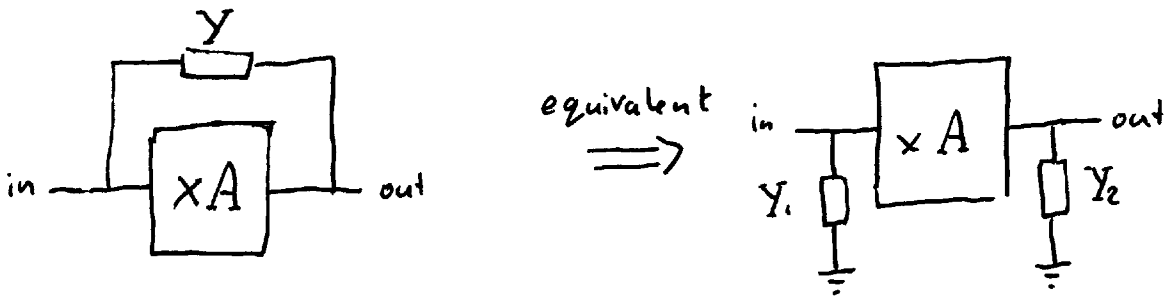
C_π is capacitance between base and emittor

C_{ce} is capacitance between collector and emittor and is normally emitted from the calculations.

Examples: $C_\pi = 25\,pF$, $C_\mu = 10\,pF$

To find the upper cut-off frequency f_H we have to find the effective resistance these capacitors see. However, there is a complication: The capacitors are connected to both the input and the output. For instance C_μ is at the input (V_B) and the output (V_C) of the transistor. This causes feedback (see next chapter). We can analyze it if we use Miller's theorem

Miller's Theorem



An admittance Y (for instance C, L or 1/R) bridging part of a circuit with voltage gain A can be decomposed into a circuit with an admittance at the entrance (Y_1) and at the exit (Y_2) of the amplifier.

$$\boxed{\begin{aligned} Y_1 &= Y(1-A) \\ Y_2 &= Y(1-1/A) \end{aligned}}$$

The admittance is amplified at the entrance and reduced (slightly) at the exit.

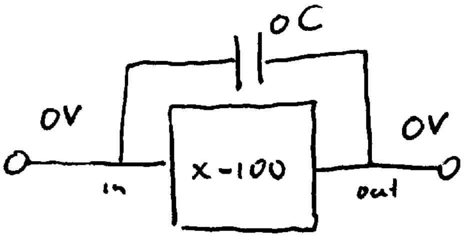
How can this be? Take the example of a capacitor C bridging an amplifier with 100 x gain. The capacitance that is felt at the entrance is the charge that is stored in the capacitor. By definition:

$$C_{eff} = \frac{dq}{dV}$$

example $C = 1\,nF$

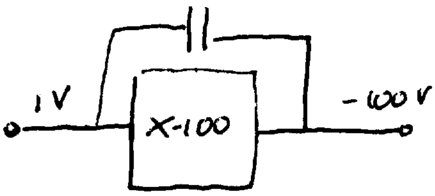
$A = -100$.

At 0 V at entrance : the exit voltage is also 0 V and the capacitor is empty



$$Q = \Delta V\, C = (V_{in} - V_{out}) \times 1\,nF = 0\ C.$$

Let us increase it to 1 V at the entrance and see how much charge will be in the capacitor, with a gain of -100 there will be -100 V at the other side. The amount of charge in the capacitor is then

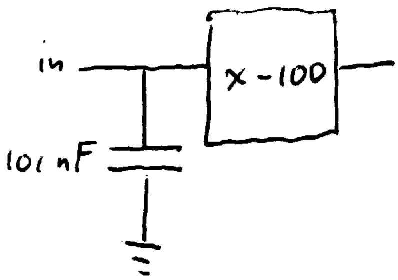


$$Q = \Delta V \cdot C = (1 - (-100)) \times 1\,nF = 101\ nC$$

Seen from the entrance, it looks like a capacitor

$$C = \frac{\overset{\text{total charge}}{Q}}{\underset{\text{input signal}}{1\,V}} = \frac{101\,nC}{1\,V} = 101\ nF$$

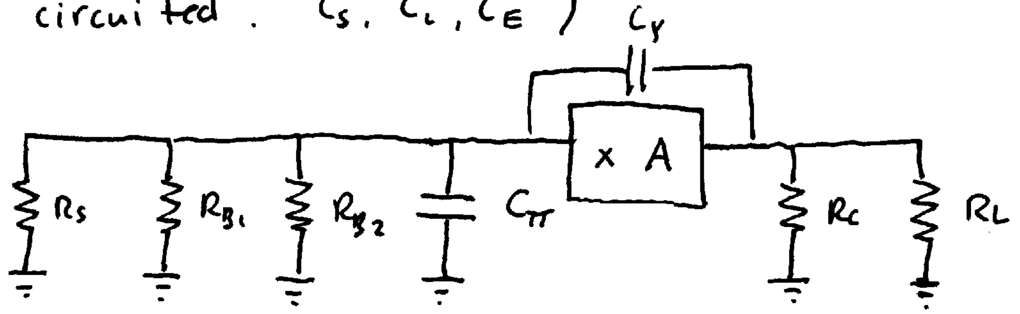
in other words, at the input it looks like the figure below.



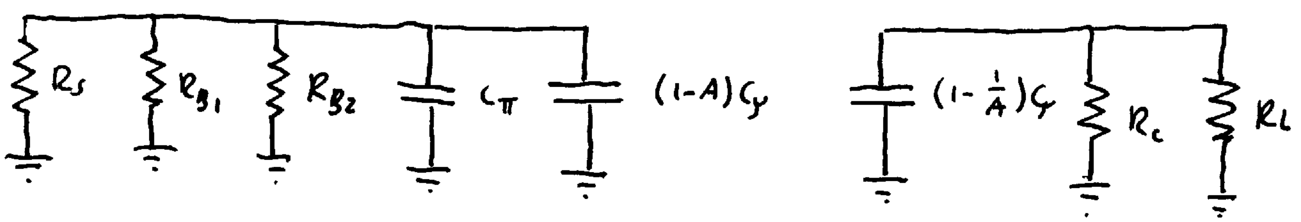
For other types of admittance we can make a similar admittance. In summary :
capacitances are multiplied at entrance. Resistances are reduced

Note that we can only use Millers theorem when the admittance does not (significantly) change the gain. It can (therefore) also not be used to determine the input and output resistances of the amplifier.
The effect on the capacitances is called the Miller effect and this influences the high-frequency response of the amplifier.

We will consider the two capacitors C_μ and C_π.
They will cauce low-pass filters
As the circuit we have (note: all other capacitors short circuited. C_S, C_L, C_E)



$\Downarrow$ MILLER



The voltage gain A from the entrance to the exit of the transistor (base-to-collector, where the capacitor is connected) is (note: R_E is bypassed)

$$A = \frac{-(R_C // R_L)}{r_e} = -\frac{(3.3\,k\Omega // 22\,k\Omega)}{20\,\Omega} = -143$$

(note the tremendous effect in gain caused by bypassing R_E. From ≈ -1 to ≈ -143)

Now we can calculate the time constants:

$$\tau_{in} = (R_S // R_{B1} // R_{B2})\,(C_\pi // (1-A)\,C_\gamma)$$

$$= (1\,k\Omega // 10\,k\Omega // 10\,k\Omega)\,(25\,pF + 144 \cdot 10\,pF)$$

$$= 833\,\Omega \cdot 1465\;10^{-12}\,F$$

$$= 1220\;ns \qquad \left(f_{H\,in} = \frac{1}{2\pi\,\tau_{in}} = 130\;kHz\right)$$

[note: the "Miller effect" for C_π is 1, because the "gain" from base to emitter is 0 (emittor is connected to ground), thus $C_M = (1-A)\,C_\pi = (1-0)\,C_\pi = C_\pi$]

$$\tau_{out} = (R_C // R_L)\,\left(\left(1-\tfrac{1}{A}\right)\cdot C_\gamma\right)$$

$$= (3.3\,k\Omega // 22\,k\Omega)\,\left(1+\tfrac{1}{143}\right)\cdot 10\,pF$$

$$= 2.87\,k\Omega \cdot 10.07\,pF$$

$$= 29\;ns \qquad \left(f_{H\,out} = \frac{1}{2\pi\,\tau_{out}} = 5.5\;MHz\right)$$

There are thus two low-pass filters. One with a time constant of 1220 ns and the other with

$\tau = 29\ ns$. It is immediately clear (the time constants are well seperated) that the first one is dominant. The cut-off frequency thus becomes

$$\omega_H = \frac{1}{\tau_{in}} = 8.2\ 10^6\ rad/s$$

$$f_H = \frac{\omega_H}{2\pi} = 130\ kHz.$$

In case the individual cut-off frequencies are not well seperated, we can use the trick of

OPEN - CIRCUIT TIME CONSTANTS TO DETERMINE HIGHER CUT-OFF FREQUENCY

- ⇒ Source to ground
- ⇒ for every capacitor causing a LPF, consider all other capacitors (of LPF's) as open circuit.
- ⇒ Determine τ_i for every capacitor in this way
- ⇒ $\tau_{tot} = \sum \tau_i$
- ⇒ $\omega_H = \frac{1}{\tau_{tot}} = \frac{1}{\sum \tau_i}$, $f_H = \frac{1}{2\pi}\omega_H$

Like for lower cut-off frequency, this is exact

In this case we would find

$$\tau_{tot} = \tau_{in} + \tau_{out} = 1220\ ns + 29\ ns = 1249\ ns$$

$$\omega_H = \frac{1}{1249\ ns} = 8.0\ 10^6\ rad/s$$

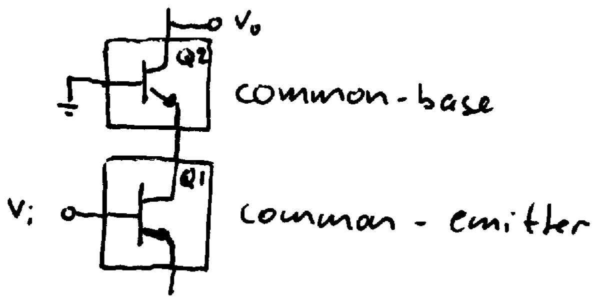
$$f_H = 127\ kHz \quad \text{(compare to dominat-pole technique)}$$

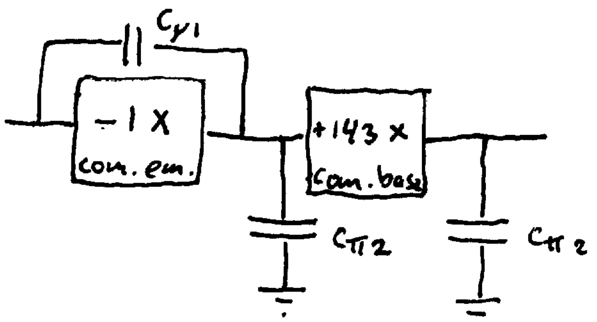
It is clear that the response is completely limited by the Miller effect. Without it we would win more-or less two decades (130 kHz to 13 MHz).

⇒ insert p 22

As we will see in the practical lectures, the sollution to this is the cascode amplifier.

A cascode amplifier is a common-emitter amplifier in series with a common-base amplifier

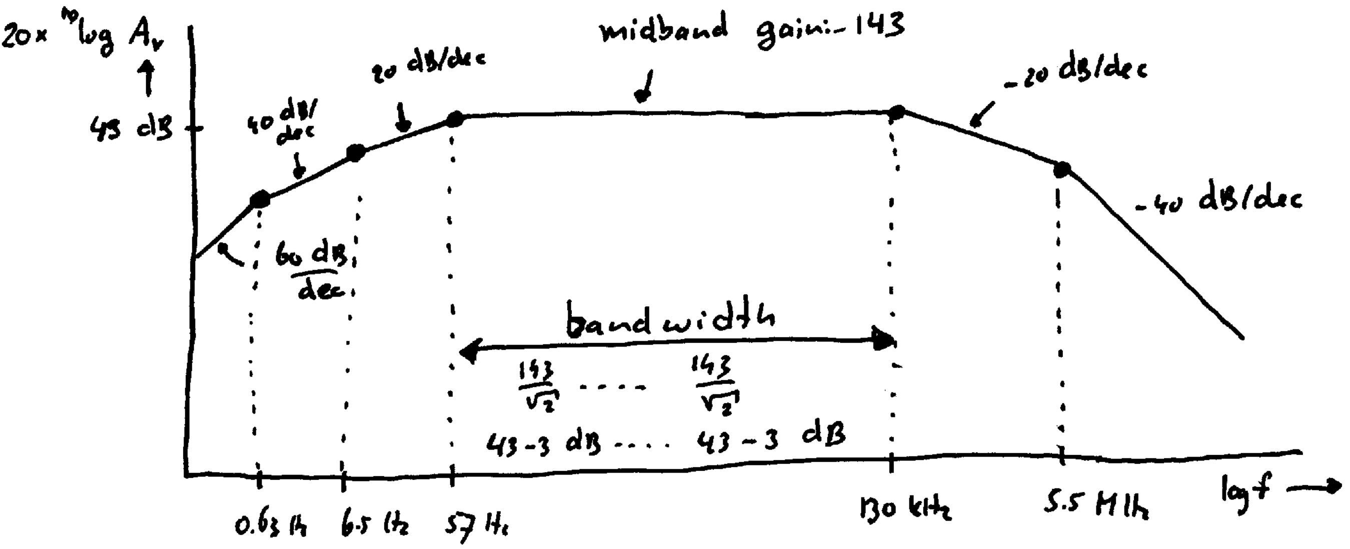


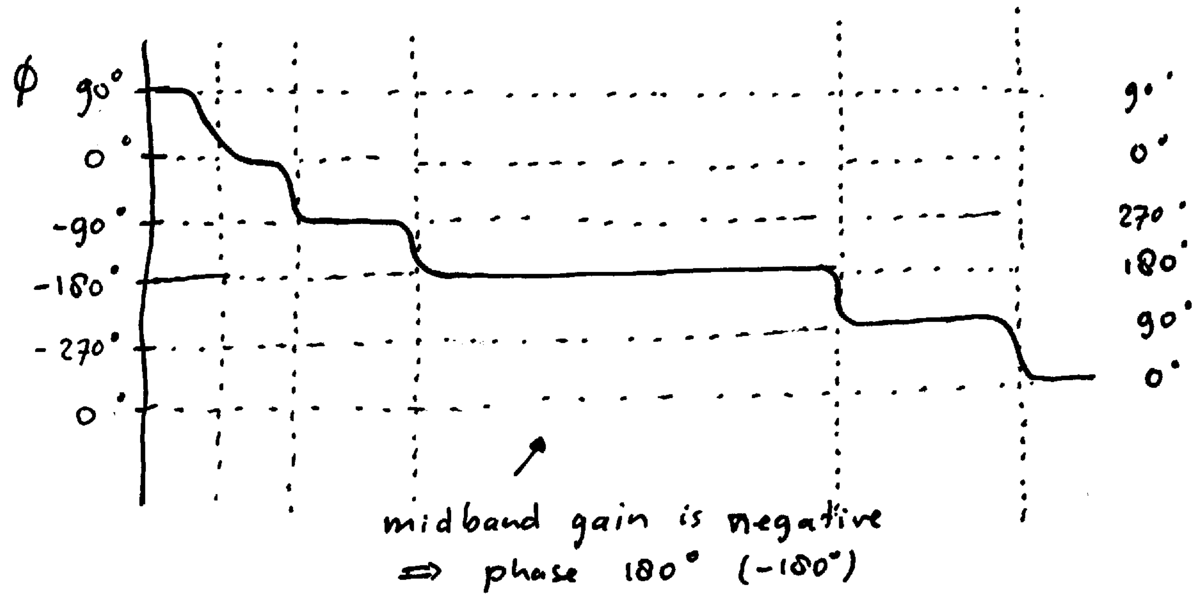


The gain of the common emitter is -1, because it is $-\frac{r_{e2}}{r_{e1}}$. The Miller effect is reduced to only a factor $1-(-1) = 2$.

The common-base stage has a high gain $\left(+\frac{R_c // R_L}{r_{e2}}\right)$ but because the base is connected to ground, the Miller effect has disappeared $(=1)$. Only $C_{\pi 2}$ at input and output.

Summary / Bode plot of the CEA





At the practical lessons we will further study the frequency response of the differential pair of chapter 1. (For more information, see Sedra's chapter 7.8)