Trabalho 2. Theoretical Values

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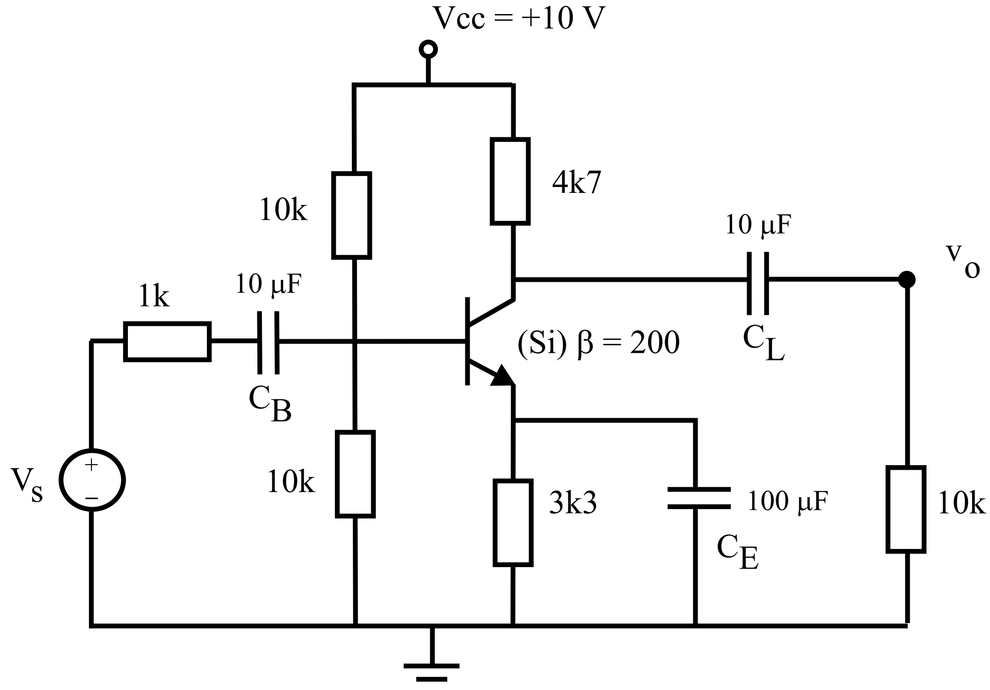
1: Common Emitter

Assume: $\beta = 200$ ($\alpha = 1$), $V_A = 120$ V, $C_\mu = 8$ pF, $C_\pi = 25$ pF



Actual values:
$R_S = 1$ kΩ
$C_S = 10$ µF
$R_{B1} = 10$ kΩ
$R_{B2} = 20$ kΩ
$R_C = 4.7$ kΩ
$C_L = 10$ µF
$R_E = 3.3$ kΩ
$C_E = 100$ µF
$R_L = 10$ kΩ
$V_{CC} = +10$ V

Bias conditions:

$V_B = +10$ V $\cdot$ 10 kΩ / (20 kΩ + 10 kΩ) = 3.33 V
$V_E = V_B - 0.7$ V = 2.63 V
$I_E = 2.63$ V / 3.3 kΩ = 0.797 mA
$V_C = 10 - 0.797\ 10^{-3} * 4700 = 6.25$ V
$r_e = 26$ mV / 0.798 mA = 32.58 Ω
$r_\pi = (\beta + 1) r_e = 6.5449$ kΩ
$r_o = 120$ V / 0.798 mA = 150 kΩ
$g_m = 1/r_e = 30.7$ mS

Gain:

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The gain is defined by the standard formula for a CE amplifier:

$$A_{\rm v} = -\frac{R_{\rm C}}{R_{\rm E} + r_{\rm e}}$$

In the intermediate frequency regime all the condensators (of the LPFs) can be considered short-circuited. Thus, the resistor $R_{\rm E}$ is bypassed and the gain increases to

$$A_{\rm v} = -\frac{R_{\rm C}}{r_{\rm e}} = -\frac{4700}{32.58} = -144.26$$

To calculate the input resistance of the transistor and the stage we also consider $R_{\rm E}$ bypassed:

$$r_{\rm in}(\text{transistor}) = (\beta + 1) r_{\rm e} = 201 \cdot 32.58\ \Omega = 6.549\ \text{k}\Omega$$

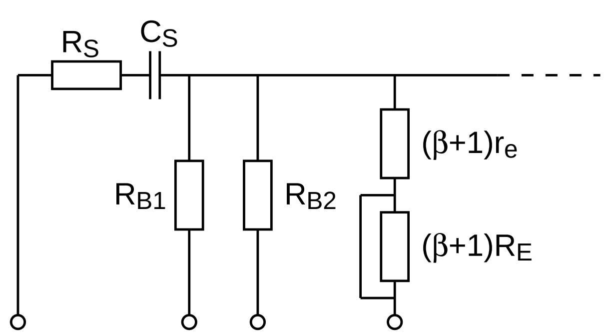
$$\begin{aligned} r_{\rm in}(\text{stage}) &= R_{\rm B1} \| R_{\rm B2} \| r_{\rm in} \\ &= 10\ \text{k}\Omega \| 20\ \text{k}\Omega \| 6.549\ \text{k}\Omega = 3.304\ \text{k}\Omega \end{aligned}$$

$$\begin{aligned} \frac{v_{\rm o}}{v_{\rm i}} &= A_{\rm v} \left[\frac{r_{\rm in}(\text{stage})}{r_{\rm S} + r_{\rm in}(\text{stage})}\right] \left[\frac{R_{\rm L}}{R_{\rm L} + R_{\rm C}}\right] \\ &= -144.26 \left(\frac{3.304}{1 + 3.304}\right) \left(\frac{10}{10 + 4.7}\right) \\ &= -144.26 \cdot 0.7677 \cdot 0.6803 = -75.33 \end{aligned}$$

$\boxed{\text{lower cut-off frequency:}}$

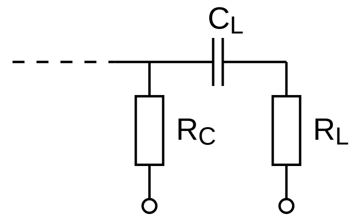
Using the short-circuit-time-constants method (all other C's are short circuited)

1: The condensator ($C_{\rm S}$) at the input is part of a high-pass filter (HPF). To calculate the cut-off frequency of this filter we have to calculate the RC time and hence we have to determine the resistance this condensator sees. Figure 2 summarizes this. The part after the collector can be neglected, because it is much larger. The resistor $R_{\rm E}$ is bypassed by the capacitor $C_{\rm E}$.



$$\begin{aligned} R &= R_{\rm S} + R_{\rm B1} \| R_{\rm B2} \| (\beta + 1) r_{\rm e} \\ &= 1\text{k}\Omega + 10\ \text{k}\Omega \| 20\ \text{k}\Omega \| 6.549\ \text{k}\Omega \\ &= 4.304\ \text{k}\Omega \\ C_{\rm S} &= 10\ \mu\text{F} \\ \tau_S &= RC = 43.04\ \text{ms} \\ f_{LS} &= \frac{1}{2\pi\tau_S} = 3.7\ \text{Hz} \end{aligned}$$

2: For the condensator at the load we have:

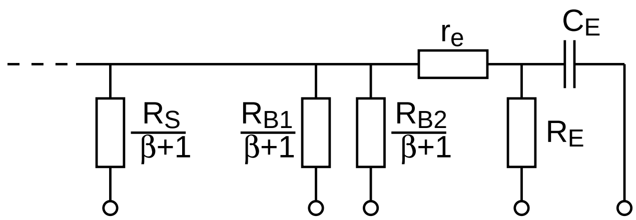


$$R = R_{\mathrm{L}} + R_{\mathrm{C}} = 10\ \mathrm{k}\Omega + 4.7\ \mathrm{k}\Omega = 14.7\ \mathrm{k}\Omega$$

(note: the resistance after the collector is again neglected). With the capacitance $C = C_{\mathrm{L}} = 10\ \mu\mathrm{F}$ we have

$$\begin{aligned}\tau_L &= RC = 147\ \mathrm{ms} \\ f_{LL} &= \frac{1}{2\pi\tau_L} = 1.08\ \mathrm{Hz}\end{aligned}$$

3: Finally, for the emittor condensator we have (everything at the collector ignored):



$$\begin{aligned}R &= R_{\mathrm{E}} \| (r_{\mathrm{e}} + \frac{R_{\mathrm{B1}}}{\beta+1} \| \frac{R_{\mathrm{B2}}}{\beta+1} \| \frac{R_{\mathrm{S}}}{\beta+1}) \\ &= 3300\ \Omega \| (32.58\ \Omega + 49.75\ \Omega \| 99.50\ \Omega \| 4.975\ \Omega) = 36.5\ \Omega \\ C &= C_{\mathrm{E}} = 100\ \mu\mathrm{F} \\ \tau_E &= RC = 3.65\ \mathrm{ms} \\ f_{LE} &= \frac{1}{2\pi\tau_E} = 43\ \mathrm{Hz}\end{aligned}$$

The cut-off frequency (f_L) is determined by the higher of the three f_{LS}, f_{LL} and f_{LE}, namely by condensator at the emittor, $f_L = f_{LE}$. More exact, $f_L = f_{LS} + f_{LL} + f_{LE} = 3.7$ Hz + 1.08 Hz + 43 Hz = 48 Hz. Below this frequency the attenuation is 20 dB / decade. The phase slowly shifts 90° and is 45° at f_L. When the frequency is further lowered it hits the second cut-off frequency (approximately f_{LS}) from where the attenuation will be 40 dB / decade, etc. Each filter introduces an attenuation of 20 dB / decade and a phase shift of 90°.

higher cut-off frequency:

Using the open-circuit-time-constants method (all other high frequency C's are open circuit).
The higher cut-off frequencies are determined by the shunt capacitances that connect the input or the output to ground. We have C_π (between base and emittor) and C_μ (between base and collector). Note that in this configuration the latter is connected between the input and the output (with negative gain) and is therefore susceptible to feedback effects (resulting in so-called Miller capacitance, C_{M}).
For the higher cut-off frequencies it is most convenient to decompose the system into an input filter and an output filter. This is allowed because the resistance between the input and the output is so high that there is virtually no contact between these points. At the input we have:

$$\begin{aligned}R &= R_{\mathrm{S}} \| R_{\mathrm{B1}} \| R_{\mathrm{B2}} \| (\beta+1) r_{\mathrm{e}} \\ &= 1\ \mathrm{k}\Omega \| 10\ \mathrm{k}\Omega \| 20\ \mathrm{k}\Omega \| 6.549\ \mathrm{k}\Omega\end{aligned}$$

$$
\begin{aligned}
&= 767.6\ \Omega \\
C &= C_\pi + C_\mathrm{M} = C_\pi + C_\mu(1 - A_\mathrm{v}) \qquad (1) \\
&= 25\ \mathrm{pF} + 8\ \mathrm{pF}(1 + 98.14) \\
&= 818.12\ \mathrm{pF} \\
\tau_i &= RC = 627.99\ \mathrm{ns} \\
f_{Hi} &= \frac{1}{2\pi\tau_i} = 253\ \mathrm{kHz}
\end{aligned}
$$

Note that for the amplification A_v we have to use the voltage amplification factor between the collector and the base because the condesator C_μ is connected between those points. It does not include the voltage devider at the source:

$$A_\mathrm{v} = -\frac{R_\mathrm{C} \| R_\mathrm{L}}{r_\mathrm{e}} = -98.14$$

At the output we have:

$$
\begin{aligned}
R &= r_\mathrm{c} \| R_\mathrm{C} \| R_\mathrm{L} \\
&= 150\ \mathrm{k}\Omega \| 4.7\ \mathrm{k}\Omega \| 10\ \mathrm{k}\Omega \\
&= 3.131\ \mathrm{k}\Omega \\
C &= C_\mu(1 - 1/A_\mathrm{v}) \\
&= 8\ \mathrm{pF} \cdot 1.0102 \\
&= 8.08\ \mathrm{pF} \\
\tau_o &= RC = 25.3\ \mathrm{ns} \\
f_{Ho} &= \frac{1}{2\pi\tau_o} = 6.3\ \mathrm{MHz}
\end{aligned}
$$

The cut-off frequency is therefore determined by the filter at the input. Using the open-circuit-time-constants method $\tau = \tau_i + \tau_o = 627.99$ ns $+ 25.3$ ns $= 655$ ns and $f_H = 1/2\pi\tau = 243$ kHz.

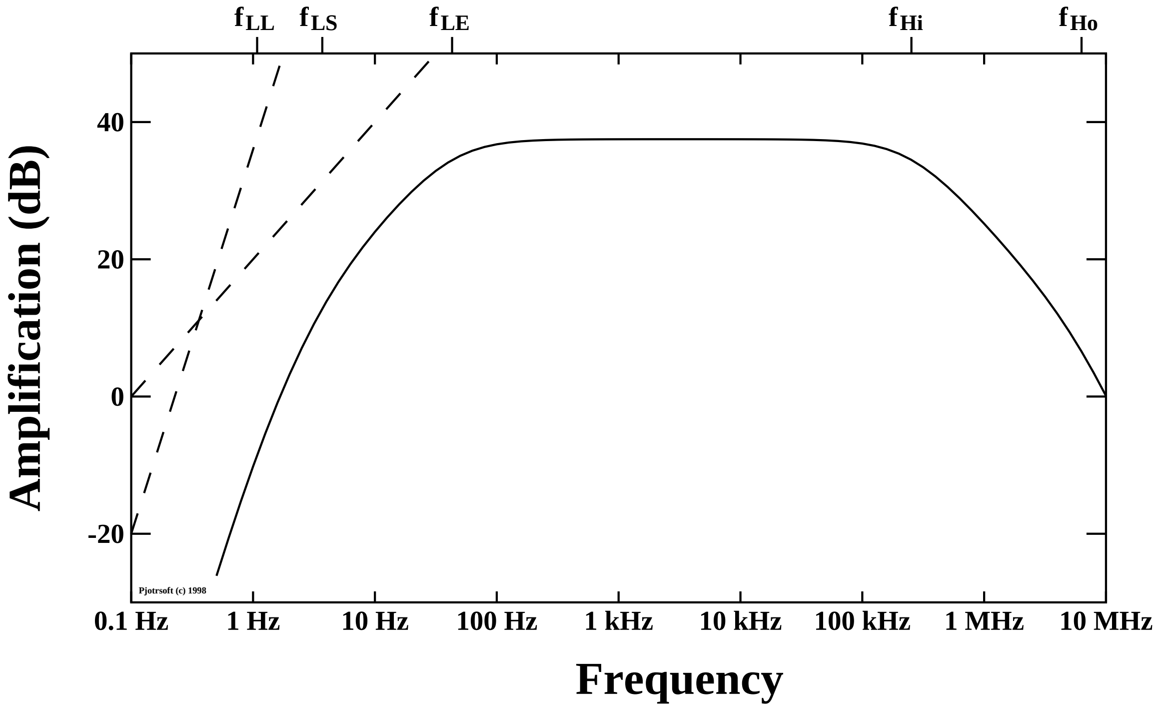
Finally, the internal cut-off frequency of the transistor is determined by (see p. 437 of Bogart)

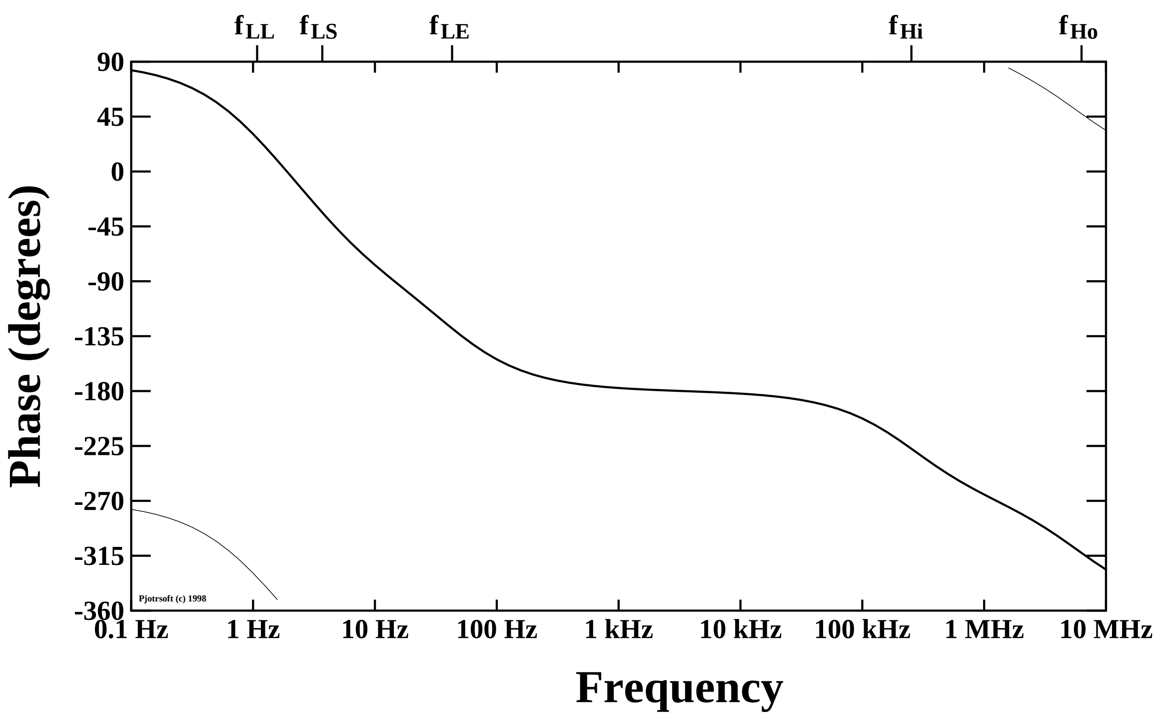
$$f_\mathrm{T} = \frac{1}{2\pi r_\mathrm{e}(C_\pi + C_\mu)} = \frac{1}{2\pi(32.58\ \Omega)(12\ \mathrm{pF})} = 407\ \mathrm{MHz}$$

(According to the specifications (datasheets), for the 2N2222 the unitary current amplification frequency is $f_\mathrm{T} = 250$ MHz.) This cut-off frequency is not important for the circuit. The gain and cut-off frequencies are summarized in the Bode plot on the next page.

phase diagram:

The overall gain of the amplifier at mid-band frequencies is negative. This is equal to a phase shift of 180°. The filter at the source with f_{LS} introduces a phase shift of +90° (see p. 33 of Sedra) at low frequencies. In fact, every HPF introduces a phase shift of 90°. On the other side of the spectrum, every LPF introduces a phase shift of −90° at high frequencies. The figures show the voltage gain (in dB) at the load and the phase relative to the source voltage:





$\boxed{\text{Without } C_E\text{:}}$

When the shunt capacitance C_{E} is pulled, the gain of the amplifier reduces to

$$A_{\mathrm{v}} = -\frac{R_{\mathrm{C}}}{r_{\mathrm{e}} + R_{\mathrm{E}}} = -\frac{4700}{32.58 + 3300} = -1.410$$

The voltage gain between the base and the collector (to which C_μ is connected; we will use this value to calculate the Miller capacitance) when a load resistance is connected is

$$A_\mathrm{v} = -1.410 \cdot \frac{R_\mathrm{L}}{R_\mathrm{L} + R_\mathrm{C}} = -1.410 \cdot \frac{10}{10 + 4.7} = -0.959$$

The voltage divider at the input:

$$\begin{aligned} r_\mathrm{in} &= (\beta + 1)(r_\mathrm{e} + R_\mathrm{E}) - 669.8 \text{ k}\Omega \\ (r_\mathrm{in})_\mathrm{stage} = r_\mathrm{in} \| R_\mathrm{B1} \| R_\mathrm{B2} &= 669.8 \text{ k}\Omega \| 10\text{k}\Omega \| 20\text{k}\Omega = 6.6 \text{ k}\Omega \\ \frac{v_\mathrm{i}}{v_\mathrm{S}} = \frac{(r_\mathrm{in})_\mathrm{stage}}{(r_\mathrm{in})_\mathrm{stage} + R_\mathrm{S}} &= \frac{6.6}{6.6 + 1} = 0.868 \end{aligned}$$

The ratio of the voltage at the output and the voltage at the input then becomes (taking into account the voltage dividers at the input and the output):

$$\frac{v_\mathrm{o}}{v_\mathrm{i}} = -0.832$$

The filter at the emittor is removed and the highest cut-off frequency is then $f_1(\mathrm{S})$ whose value is also slightly changed by the elimination of the shunt capacitance:

$$\begin{aligned} R &= R_\mathrm{S} + R_\mathrm{B1} \| R_\mathrm{B2} \| (\beta + 1)(r_\mathrm{e} + R_\mathrm{E}) \\ &= 7.601 \text{ k}\Omega \\ C &= 10\ \mu\text{F} \\ \tau_S &= RC = 76.01 \text{ ms} \\ f_{LS} &= \frac{1}{2\pi\tau_S} = 2.09 \text{ Hz} \\ f_{LL} &= f_{LL} = 1.08 \text{ Hz} \end{aligned}$$

For the higher cut-off frequency.
1: At the entrance of the transistor:

$$\begin{aligned} R &= R_\mathrm{S} \| R_\mathrm{B1} \| R_\mathrm{B2} \| (\beta + 1)(R_\mathrm{E} + r_\mathrm{e}) \\ &= 1\text{k}\Omega \| 10\text{k}\Omega \| 20\text{k}\Omega \| 669.8\text{k}\Omega = 868\ \Omega \\ C &= C_\pi + (1 - A_\mathrm{v})C_\mu = 25 \text{ pF} + (1 + 0.959) \cdot 8 \text{ pF} = 40.7 \text{ pF} \\ \tau_i &= RC = 35.3 \text{ ns} \\ f_{Hi} &= \frac{1}{2\pi\tau_i} = 4.5 \text{ MHz} \end{aligned}$$

2: At the exit of the transistor:

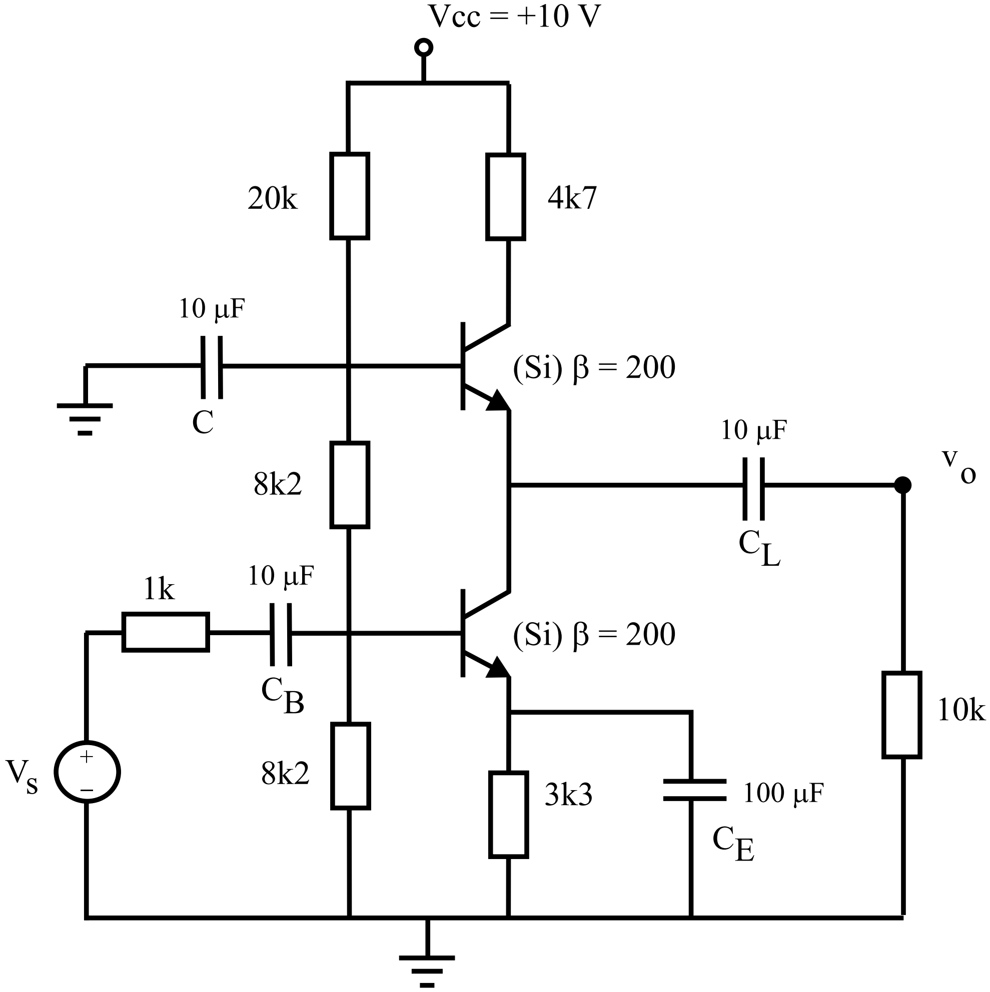
$$\begin{aligned} R &= R_\mathrm{L} \| R_\mathrm{C} = 3.131 \text{ k}\Omega \\ C &= (1 - 1/A_\mathrm{v})C_\mu = (1 + 1/0.959) \cdot 8 \text{ pF} = 16.3 \text{ pF} \\ \tau_o &= RC = 51.0 \text{ ns} \\ f_{Ho} &= \frac{1}{2\pi\tau_o} = 3.12 \text{ MHz} \end{aligned}$$

3: internal filter:

$$f'_T = f_T = 407 \text{ MHz}$$

The cut-off frequency is determined by the exit of the transistor.

2: Cascode



Actual values:
$R_{\mathrm{S}} = 1\ \mathrm{k}\Omega$
$C_{\mathrm{S}} = 10\ \mu\mathrm{F}$
$R_{\mathrm{B1}} = 8.2\ \mathrm{k}\Omega$
$R_{\mathrm{B2}} = 8.2\ \mathrm{k}\Omega$
$R_{\mathrm{B3}} = 20\ \mathrm{k}\Omega$
$R_{\mathrm{C}} = 4.7\ \mathrm{k}\Omega$
$C_{\mathrm{L}} = 10\ \mu\mathrm{F}$
$R_{\mathrm{E}} = 3.3\ \mathrm{k}\Omega$
$C_{\mathrm{E}} = 100\ \mu\mathrm{F}$
$R_{\mathrm{L}} = 10\ \mathrm{k}\Omega$
$V_{\mathrm{CC}} = 15\ \mathrm{V}$
$C_{\mathrm{B}} = 10\ \mu\mathrm{F}$

Bias conditions:

$V_{\mathrm{B1}} = +15\ \mathrm{V} \cdot 8.2\ \mathrm{k}\Omega\ /\ (20\ \mathrm{k}\Omega + 8.2\ \mathrm{k}\Omega + 8.2\ \mathrm{k}\Omega) = 3.38\ \mathrm{V}$
$V_{\mathrm{E1}} = V_{\mathrm{B}} - 0.7\ \mathrm{V} = 2.68\ \mathrm{V}$
$I_{\mathrm{E1}} = 2.68\ \mathrm{V}\ /\ 3.3\ \mathrm{k}\Omega = 0.8119\ \mathrm{mA}$ (3.3 kΩ choosen so that I_{E1} will be approximately 0.8 mA)
$I_{\mathrm{C2}} = I_{\mathrm{E2}} = I_{\mathrm{C1}} = I_{\mathrm{E1}}$ (α=1)
$V_{\mathrm{C2}} = +15 - R_{\mathrm{C}} I_{\mathrm{C2}} = 11.18\ \mathrm{V}$
$r_{\mathrm{e}} = 26\ \mathrm{mV}\ /\ 0.8119\ \mathrm{mA} = 32.024\ \Omega$
$g_{\mathrm{m}} = 1/r_{\mathrm{e}} = 31.2\ \mathrm{mS}$
$r_\pi = (\beta + 1) r_{\mathrm{e}} = 6.437\ \mathrm{k}\Omega$
$r_{\mathrm{o}} = 120\ \mathrm{V}\ /\ 0.8119\ \mathrm{mA} = 150\ \mathrm{k}\Omega$
The output resistance of Q1 is r_o. The output resistance of Q2 is much higher (reasoning is similar to current source of TP1)

Gain:

$$\begin{aligned}(r_{\mathrm{in}})_{\mathrm{stage}} &= (\beta+1) r_{\mathrm{e1}} \| R_{\mathrm{B1}} \| R_{\mathrm{B2}} \\ &= 6.437\ \mathrm{k}\Omega \| 8.2\ \mathrm{k}\Omega \| 8.2\ \mathrm{k}\Omega \\ &= 2.505\ \mathrm{k}\Omega\end{aligned}$$

$$
\begin{aligned}
(r_\mathrm{o})_\mathrm{stage} &= R_\mathrm{C} \| r_{\mathrm{out},2} = R_\mathrm{C} \\
&= 4.7\ \mathrm{k}\Omega \\
A_\mathrm{v}^1 &= -\frac{r_{\mathrm{out},1} \| r_\mathrm{e2}}{r_\mathrm{e1}} = -\frac{r_\mathrm{e2}}{r_\mathrm{e1}} = -1 \\
A_\mathrm{v}^2 &= \frac{R_\mathrm{C} \| r_{\mathrm{out},2}}{r_\mathrm{e2} \| r_\mathrm{c1}} = \frac{R_\mathrm{C}}{r_\mathrm{e2}} = \frac{4700}{32.024} = +147.76 \\
A_\mathrm{v} &= A_\mathrm{v}^1 \cdot A_\mathrm{v}^2 = -1 \cdot 147.76 = -147.76 \\
\frac{v_\mathrm{L}}{v_\mathrm{S}} &= \frac{r_\mathrm{in}}{r_\mathrm{in} + r_\mathrm{S}} \cdot A_\mathrm{v} \cdot \frac{r_\mathrm{L}}{r_\mathrm{L} + r_\mathrm{o}} \\
&= \left(\frac{2505}{1000 + 2505}\right)(-146.76)\left(\frac{10000}{10000 + 4700}\right) \\
&= 0.7147 \cdot (-146.764) \cdot 0.6803 \\
&= -71.36
\end{aligned}
$$

At low frequencies nothing much changes, though there is an extra filter caused by the condensator C_B:

$$
\begin{aligned}
C &= C_\mathrm{S} = 10\ \mu\mathrm{F} \\
R &= R_\mathrm{S} + R_\mathrm{B1} \| R_\mathrm{B2} \| (\beta + 1) r_\mathrm{e} = 1 + 8.2 \| 8.2 \| 6.437 = 3.505\ \mathrm{k}\Omega \\
\tau_S &= RC = 35.05\ \mathrm{ms} \\
f_{LS} &= 4.54\ \mathrm{Hz}
\end{aligned}
$$

$$
\begin{aligned}
C &= C_\mathrm{B} = 10\ \mu\mathrm{F} \\
R &= R_\mathrm{B3} \| (R_\mathrm{B2} + R_\mathrm{S} \| R_\mathrm{B1} \| (\beta + 1) r_\mathrm{e}) = 20 \| (8.2 + 1 \| 8.2 \| 6.437) = 6.199\ \mathrm{k}\Omega \\
\tau_B &= RC = 61.99\ \mathrm{ms} \\
f_{LB} &= 2.57\ \mathrm{Hz}
\end{aligned}
$$

$$
\begin{aligned}
C &= C_\mathrm{E} = 100\ \mu\mathrm{F} \\
R &= R_\mathrm{E} \| \left(r_\mathrm{e} + \frac{R_\mathrm{B1} \| R_\mathrm{B2} \| R_\mathrm{S}}{\beta + 1}\right) = 3.3 \| \left(0.032024 + \frac{8.2 \| 8.2 \| 1}{201}\right) = 35.63\ \Omega \\
\tau_E &= RC = 3.563\ \mathrm{ms} \\
f_{LE} &= 44.67\ \mathrm{Hz}
\end{aligned}
$$

$$
\begin{aligned}
C &= C_\mathrm{L} = 10\ \mu\mathrm{F} \\
R &= R_\mathrm{C} + R_\mathrm{L} = 4.7 + 10 = 14.7\ \mathrm{k}\Omega \\
\tau_L &= RC = 147\ \mathrm{ms} \\
f_{LL} &= 1.08\ \mathrm{Hz}
\end{aligned}
$$

The cut-off frequency is the largest of the four: 45 Hz. Or using the short-circuit-time-constants theory: $f_L = \sum_i f_i = 53$ Hz.
The advantage is in the high frequencies, namely the virtual elimination of the Miller capacitance because the capacitance C_μ at the entrance of the stage is connected to two points with a voltage gain close to -1 as described above. The Miller capacitance therefore reduces to

$$
C_\mathrm{M} = C_\mu \left(1 - A_\mathrm{v}\right) = 8\ \mathrm{pF}\left(1 - (-1)\right) = 16\ \mathrm{pF}
$$

and the total capacitance becomes

$$C = C_\pi + C_\mathrm{M} = 25\ \mathrm{pF} + 16\ \mathrm{pF} = 41\ \mathrm{pF}$$

The resistance is approximately the same as in Equation 1.

$$\begin{aligned}
R &= R_\mathrm{S} \| R_\mathrm{B1} \| R_\mathrm{B2} \| (\beta+1) r_\mathrm{e} = 1 \| 8.2 \| 8.2 \| 6.537 = 714.7\ \Omega \\
\tau_i = RC &= 29.3\ \mathrm{ns} \\
f_{Hi} &= 5.43\ \mathrm{MHz}
\end{aligned}$$

At the output the capacitor C_μ ow has one leg connected to ground (V_B2) and the Miller effect disappears (1-0 = 1)

$$\begin{aligned}
R &= R_\mathrm{C} \| R_\mathrm{L} = 10 \| 4.7 = 3.197\ \mathrm{k}\Omega \\
C &= C_\mu = 8\ \mathrm{pF} \\
\tau &= RC = 25.6\ \mathrm{ns} \\
f_{Ho} &= 6.2\ \mathrm{MHz}
\end{aligned}$$

In-between the two transistors we have another filter:

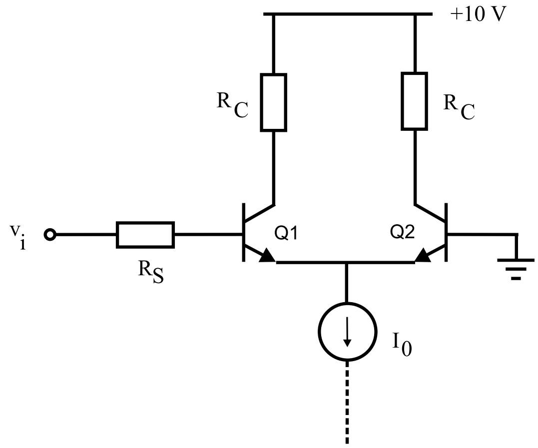
$$\begin{aligned}
C &= C_{\pi 2} + (1 - (-1/1)) C_{\mu 1} = 41\ \mathrm{pF} \\
R &= r_\mathrm{e} = 32.58\ \Omega \\
\tau &= RC = 1.34\ \mathrm{ns} \\
f_{Hm} &= 119\ \mathrm{MHz}
\end{aligned}$$

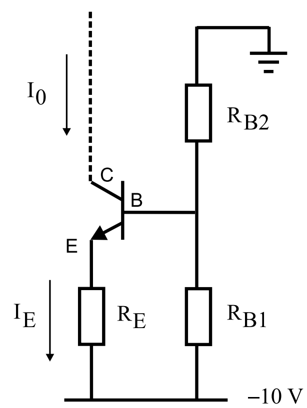
Using the theory of open-circuit time constants we find a total time constant of

$$\begin{aligned}
\tau = 29.3\ \mathrm{ns} + 25.6\ \mathrm{ns} + 1.34\ \mathrm{ns} &= 56.2\ \mathrm{ns} \\
f_\mathrm{H} = 1/2\pi\tau &= 2.8\ \mathrm{MHz}
\end{aligned}$$

Conclusion: there is a significant increase in high-frequency response by approximately one decade, basically caused by the elimination of the Miller effect at the entrance.

3: Differential Pair with Current Source





Actual values:
$R_\mathrm{C} = 3.3$ kΩ
$R_\mathrm{S} = 1$ kΩ
$I_0 = 3$ mA

bias:
$I_0 = 3$ mA, $I_\mathrm{E} = 1.5$ mA. $r_\mathrm{e} = 26$ mV / 1.5 mA $= 17.33\ \Omega$. $r_\pi = 201$ x $17.3\ \Omega = 3.48$ kΩ. $V_{o2} = V_{o1} = 10$ V - 1.5 mA x 3.3 kΩ $= 5.05$ V. $r_\mathrm{in} = 2$ x $r_\pi = 7.0$ kΩ.

gain:

$$
\begin{aligned}
A_\mathrm{v}^\mathrm{l} &= -\frac{\alpha R_\mathrm{C}}{2r_\mathrm{e}} = -95.4 \\
A_\mathrm{v}^\mathrm{r} &= +95.4 \\
\frac{v_\mathrm{o}}{v_\mathrm{s}} &= \frac{r_\mathrm{in}}{r_\mathrm{in} + R_\mathrm{S}} A_\mathrm{v} = \frac{(\beta+1)2r_\mathrm{e}}{(\beta+1)2r_\mathrm{e} + R_\mathrm{S}} A_\mathrm{v} = 0.8742 \cdot 95.2 = +83.4
\end{aligned}
$$

high-frequencies:
The high-frequency response is not improved compared to a single-stage amplifier. At the input exists $C_\pi = 25$ pF and $C_\mu = 8$ pF. Both suffer from the Miller effect and are therefore multiplied by the respective voltage gains between the two points that are bridged by the capacitance. From Trabalho 1 we know that in differential mode, the gain between base and collector of the left transistor (Q1) is given by

$$
\begin{aligned}
A_\mathrm{v} &= -\frac{\alpha R_\mathrm{C}}{2r_\mathrm{e}} \\
&= -\frac{1 \cdot 3300}{2 \cdot 17.33} \\
&= -95.2
\end{aligned}
$$

The gain felt by C_π is exactly 0.5. The total capacitance at the input is therefore

$$
\begin{aligned}
C &= (1-0.5)C_\pi + C_\mathrm{M} = 0.5C_\pi + C_\mu\,(1 - A_\mathrm{v}) \\
&= 12.5\text{ pF} + 8\text{ pF}\,(1 - (-95.4)) = 783.7\text{ pF}
\end{aligned}
$$

The resistance-to-ground at the input is

$$
R = R_\mathrm{S} \| (\beta+1)2r_\mathrm{e}
$$

$$
\begin{aligned}
&= 1\ k\Omega \| 201 \cdot 17.33\ \Omega = 777\ \Omega \\
\tau_i &= RC = 608.9 \text{ ns} \\
f_{Hi} &= \frac{1}{2\pi\tau_i} = 261 \text{ kHz}
\end{aligned}
$$

At the output (collector of right transistor Q2) we have $C_\mu = 8$ pF to the base. Without any Miller effects, because the gain between emittor and base is zero. The total capacitance is therefore

$$C = C_\mu = 8 \text{ pF}$$

At the output the resistance is $R_\mathrm{C} \| r_{\mathrm{out},2} \approx R_\mathrm{C} = 3.3$ kΩ. Therefore

$$
\begin{aligned}
\tau_o &= RC = 26.4 \text{ ns} \\
f_{Ho} &= \frac{1}{2\pi\tau_o} = 6.0 \text{ MHz}
\end{aligned}
$$

Note: with a load resistance of 10 kΩ, the equivalent resistance at the output would be 2.48 kΩ and the frequency would further increase to 8.0 MHz.
The internal cut-off frequency remains at

$$f_T = f_T = 407 \text{ MHz}$$

$\boxed{\text{low frequencies:}}$

Since there are no coupling capacitors at the entrance or the exit, there is no theoretical lower cut-off frequency. If we *do* introduce them (at the source and the load only) we can say the following: Since there is no need for any bypass condensator at the emittor we can expect an improvement of the lower frequency response, since in the single-stage amplifiers the lower-frequency response is limited by this condensator (see first part). There still remain the filters at the source and at the load. For the source we have $C_\mathrm{S} = 10$ μF and the resistance is here $R_\mathrm{S} + (\beta + 1)2r_\mathrm{e} = 4.48$ kΩ. The cut-off frequency is then $f_{LS} = 3.6$ Hz. At the load the resistance is $R_\mathrm{L} + R_\mathrm{C} \| r_\mathrm{o} \approx 4.3$ kΩ and with a load-condensator of 10 μF this gives a cut-off frequency of $f_{LL} = 3.7$ Hz.

In conclusion: For a differential pair we see a significant improvement of the low-frequency performance compared to a single-stage amplifier (about one decade), mainly due to the elimination of the emittor condensator.

FURTHER INFORMATION:

Chapter 10 of "Electronic Devices and Circuits", T. F. Bogart, Jr., 4th edition, Prentice Hall, 1997.
Chapter 7 of "Microelectronic Circuits", A. S. Sedra, K. C. Smith, 2nd edition, Oxford University Press, 1982.