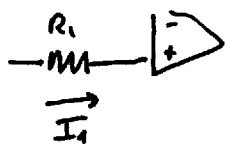


Question 1

a) $A = \infty \Rightarrow V_p = V_n$ or saturation
 $r_i = \infty, r_o = 0$

b)

• $I_p = 0$ (because $r_i = \infty$)

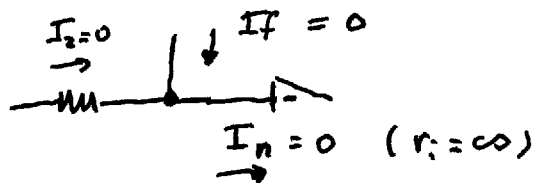


• Therefore $I_1 = 0$

• That means $V_p = V_i - I_1 R_1 = V_i$

• That means $V_n = V_i$ (rule 1 above)

• If $V_n = V_i$ then R_2 has no current. That means that I_f is zero (using Kirchoff Law)

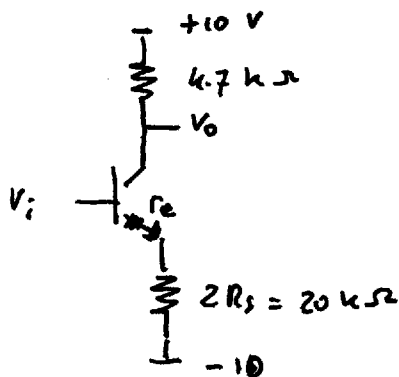


• If $I_f = 0$ then $\Delta V_f = 0 \Rightarrow V_o = V_n - I_f R_f = V_i$

$\Rightarrow V_o = V_i$

Question 2

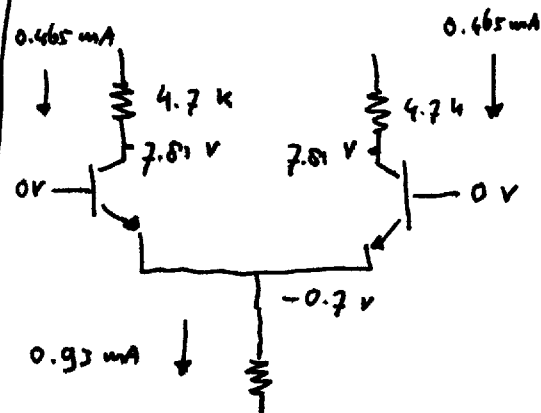
common mode gain



$$A_v^{cm} = \frac{4.7 \text{ k}\Omega}{20 \text{ k}\Omega + 53.8 \Omega} \left(\frac{R_c}{2R_s + r_e} \right)$$

$$= -0.234$$

Bias:

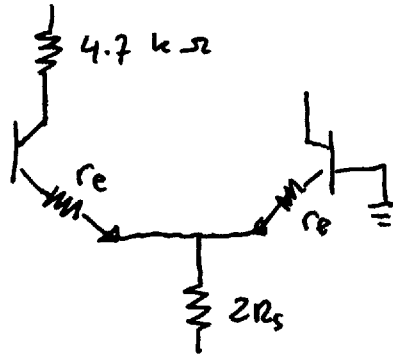


$$r_e = \frac{25 \text{ mV}}{0.465 \text{ mA}} = 53.8 \Omega$$

$$r_o = \frac{1000 \text{ V}}{0.465 \text{ mA}} = 2.1 \text{ M}\Omega$$

$$\beta \gg 1, I_c = I_E$$

Differential mode gain

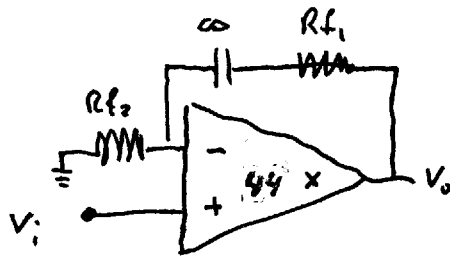


$$A_v^{dm} = + \frac{R_c}{r_e + r_e \parallel 2R_s} \approx$$

$$\approx + \frac{R_c}{2r_e} = +43.68$$

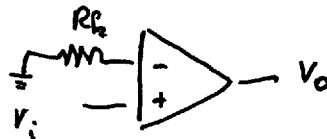
$$CMRR = \left| \frac{A_v^{dm}}{A_v^{cm}} \right| = \frac{43.68}{0.234} = 187$$

Question 3

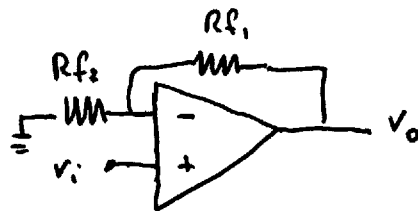


The circuit is equal to the opamp circuit

For DC the ∞ is open circuit and we get our 44 x amplifier back



For any other frequency ∞ is a short circuit and the circuit becomes



$$\beta = - \frac{R_{f2}}{R_{f1} + R_{f2}} = -0.5$$

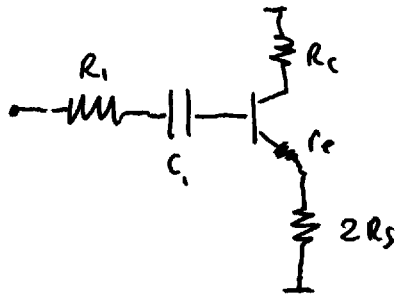
$$\frac{V_o}{V_i} = \frac{A}{1 - A\beta} = \frac{187}{1 + 187 \times 0.5} = 1.98$$

Question 4

* Common mode: The feedback capacitor C_{cc} has no effect!! Since both sides of the capacitor always have the same voltage (because of symmetry) the charge in the capacitor is zero: $Q = C \cdot \Delta V = 0$.

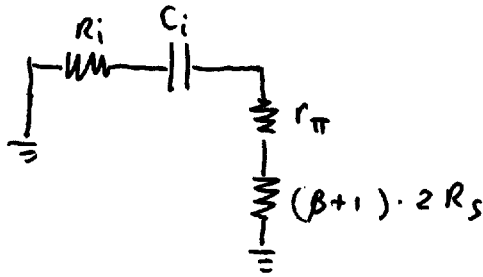
If $Q = 0$, $I = \frac{dQ}{dt} = 0$, always and the capacitor might as well not have been there at all.

We get the following circuit in CM:



C_i is part of a high-pass filter (HPF).

The equivalent resistance is given by



$$R_{eff} = R_i + r_{\pi} + (\beta + 1) \cdot 2R_s$$

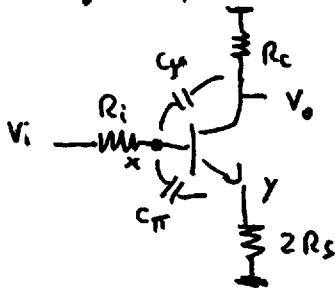
$$= 1 \text{ k}\Omega + (100) \cdot (20 \text{ k}\Omega + 53.8 \Omega)$$

$$= 2.0 \text{ M}\Omega$$

$$\tau_L = R_{eff} \cdot C_i = 2 \text{ M}\Omega \cdot 1 \mu\text{F} = 2 \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} = 80 \text{ mHz}$$

* High frequency CM: The capacitor C_i is short circuit.



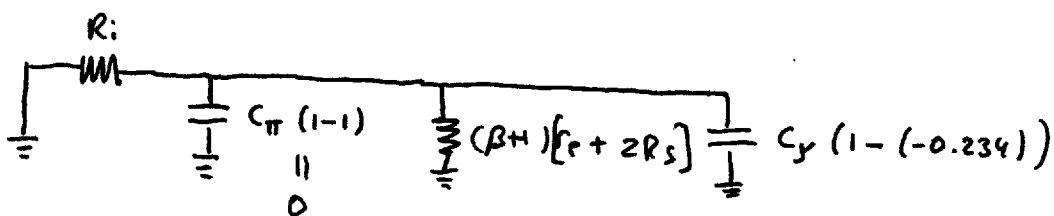
The gain between V_x and V_o is

$$-0.234 \quad (\text{for Miller effect})$$

The gain between V_x and $V_y \approx 1$

We get at the entrance the

following equivalent circuit:



$$C_{eff} = C_{\pi} \times (1 - (-0.234)) + C_{\mu} (1 - (-0.234))$$

$$= 12.34 \text{ pF}$$

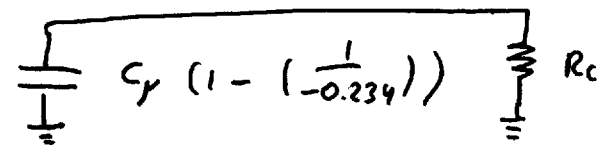
$$R_{eff} = R_i \parallel (\beta + 1)(r_e + 2R_s)$$

$$= 1 \text{ k}\Omega \parallel 2 \text{ M}\Omega \approx 1 \text{ k}\Omega$$

$$\tau_{H1} = R_{eff} \times C_{eff} = 12.34 \text{ ns}$$

$$f_{H1} = \frac{1}{2\pi\tau_{H1}} = 12.9 \text{ MHz}$$

At the exit we get following equivalent circuit:



$$C_{eff} = C_{\mu} (1 - (-0.234)) = 52.7 \text{ pF}$$

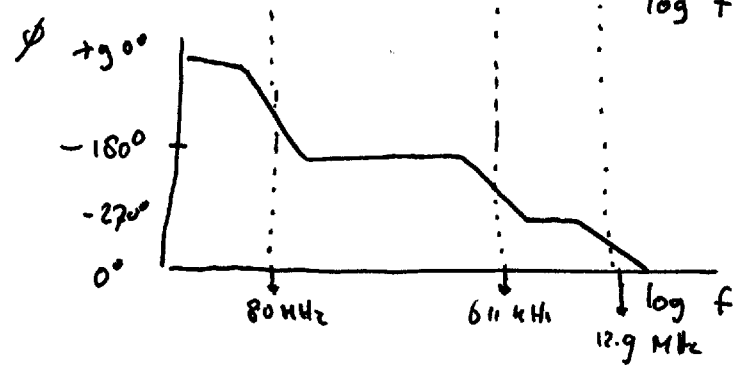
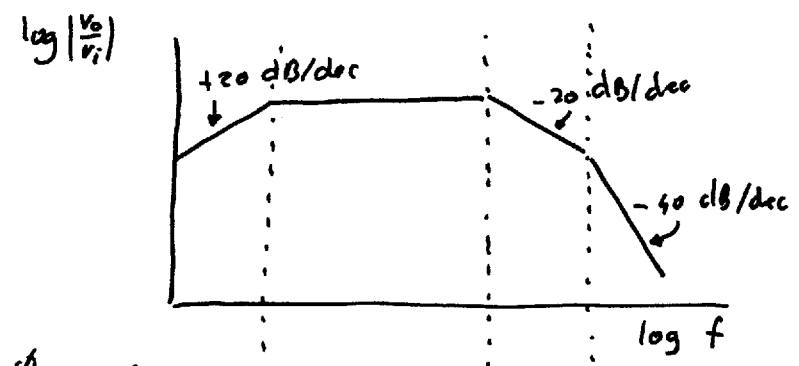
$$R_{eff} = R_c = 4.7 \text{ k}\Omega$$

$$\tau_{H2} = R_{eff} \times C_{eff} = 2.48 \times 10^{-7} \text{ s}$$

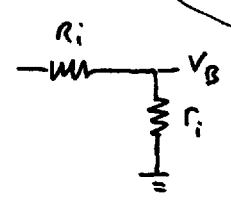
$$f_{H2} = \frac{1}{2\pi\tau_{H2}} = 642 \text{ kHz}$$

$$\tau_{total} = \tau_{H1} + \tau_{H2} = 2.60 \times 10^{-7} \text{ s}$$

$$f_{Htotal} = \frac{1}{2\pi\tau_{total}} = 611 \text{ kHz}$$



total gain \Rightarrow
 $-0.234 \times \left(\frac{r_i}{r_i + R_i} \right) \approx -1$



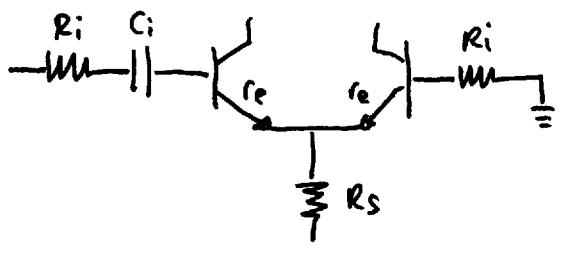
$$r_i = 2 \text{ M}\Omega = (\beta + 1)(r_e + 2R_s)$$

$$A_v = -0.234$$

* Differential mode : low frequencies

The two C_i 's are HPF. We will use the short-circuit time constants idea to find the two τ 's :

Left side : (C_i right is short circuit) :



$$R_{eff} = R_i + (\beta + 1) \left[r_e + R_s \parallel \left(r_e + \frac{R_i}{\beta + 1} \right) \right]$$

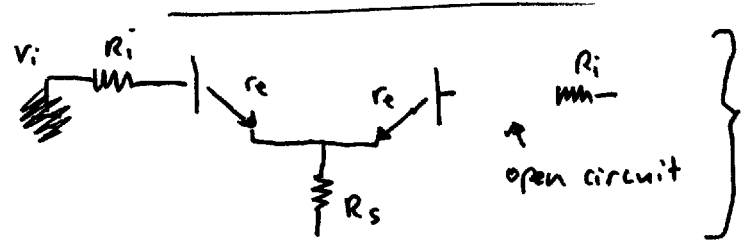
$$\approx 2R_i + 2(\beta + 1)r_e$$

$$= 2k\Omega + 107.6\Omega$$

$$\tau_L = R_{eff} \times C_i = 2.1 k\Omega \times 10 \mu F = 2.1 ms$$

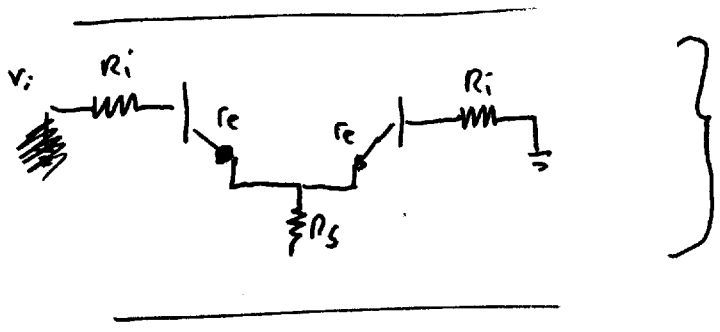
$$f_L = 75.5 Hz$$

Right side : (C_i left is short circuit) : This is also a HPF because :



low frequencies

$$A_v \approx \frac{R_c}{R_s} \approx 0$$

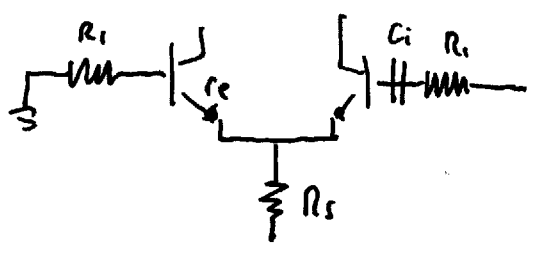


High frequencies :

$$A_v \approx \frac{R_c}{r_e + R_s \parallel \left(r_e + \frac{R_i}{\beta + 1} \right)}$$

$$\approx \frac{R_c}{2r_e} \text{ is large}$$

HPF behavior

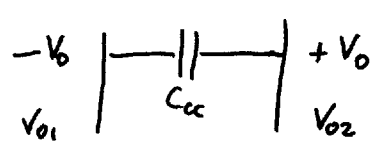


R_{eff} is equal to C_i of left side

$$\Rightarrow \tau_L = 2.1 ms, f_L = 75.5 Hz$$

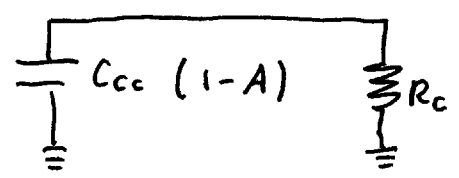
$$\tau_{total} = \left(\frac{1}{\tau_L} + \frac{1}{\tau_L} \right)^{-1} = 1.05 ms, f_{Ltotal} = f_L + f_L = 151 Hz$$

* Differential mode : High frequencies . The C_i 's are already short circuit. The capacitor C_{cc} is also cutting at high frequencies . If the outputs $-V_o$ and $+V_o$ are



Shorted (at high frequencies a capacitor is a short circuit), the outputs are both 0. (the only value that follows the symmetry rules $V_{o1} = V_{o2}$ and $V_{o1} = -V_{o2}$)

At the relevant output we have



Output resistance r_o of transistor can be ignored.

A is the gain between $V_{o2} \rightarrow V_{o1}$. $A = -1$

$$C_M = 1 \text{ nF} \times (1 - (-1)) = 2 \text{ nF}$$

$$R_C = R_c = 4.7 \text{ k}\Omega$$

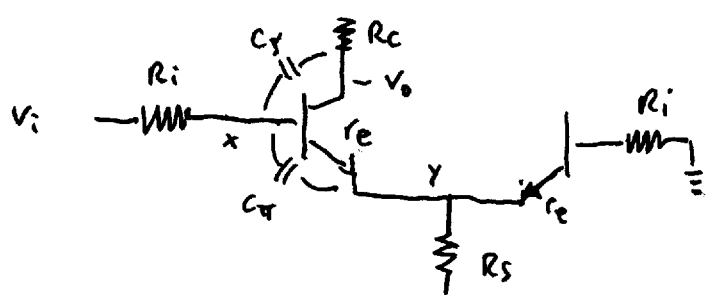
$$\tau_{CC} = R_c \times C_M = 9.4 \text{ }\mu\text{s}$$

$$f_{CC} = 16.9 \text{ kHz}$$

The other time constants can be found as before

(note : due to our way of calculating, using open-circuit time constants, C_{cc} is now absent):

At the entrance :



The gain between V_x and V_o is -44 (Question 2). For Miller effect of C_y

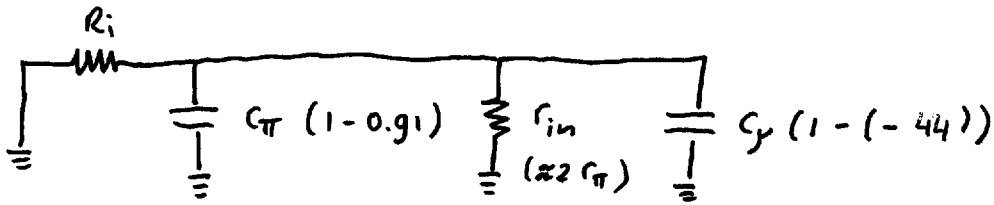
The gain between V_x and $V_y \sim$

$$\left\{ r_{in} \approx 2r_{\pi}, r_{in} = (\beta+1) \left[r_e + R_s \parallel \left(r_e + \frac{R_1}{\beta+1} \right) \right] \right\}$$

$$\frac{2r_{\pi}}{R_i + 2r_{\pi}} = 0.91$$

(7)

We get the following equivalent circuit



$$C_{eff} = C_\pi (1-0.91) + C_y (1-(-44))$$

$$= 10 \text{ pF} \times 0.09 + 10 \text{ pF} \times 45 = 0.45 \text{ nF}$$

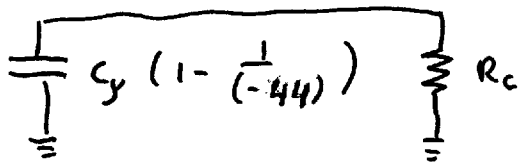
$$R_{eff} = R_i \parallel r_{in}$$

$$= 1 \text{ k}\Omega \parallel 10.7 \text{ k}\Omega = 0.91 \text{ k}\Omega$$

$$\tau_{in} = C_{eff} \times R_{eff} = 0.41 \text{ }\mu\text{s}$$

$$f_{in} = \frac{1}{2\pi \tau_{in}} = 389 \text{ kHz}$$

At the exit :



$$\tau_{out} = 10 \text{ pF} \times (1 + \frac{1}{44}) \times 4.7 \text{ k}\Omega = 48 \text{ ns}$$

$$f_{out} = \frac{1}{2\pi \tau_{out}} = 3.3 \text{ MHz}$$

$$\tau_{total} = \tau_{cc} + \tau_{in} + \tau_{out} = 9.4 \text{ }\mu\text{s} + 0.41 \text{ }\mu\text{s} + 48 \text{ ns} = 9.9 \text{ }\mu\text{s}$$

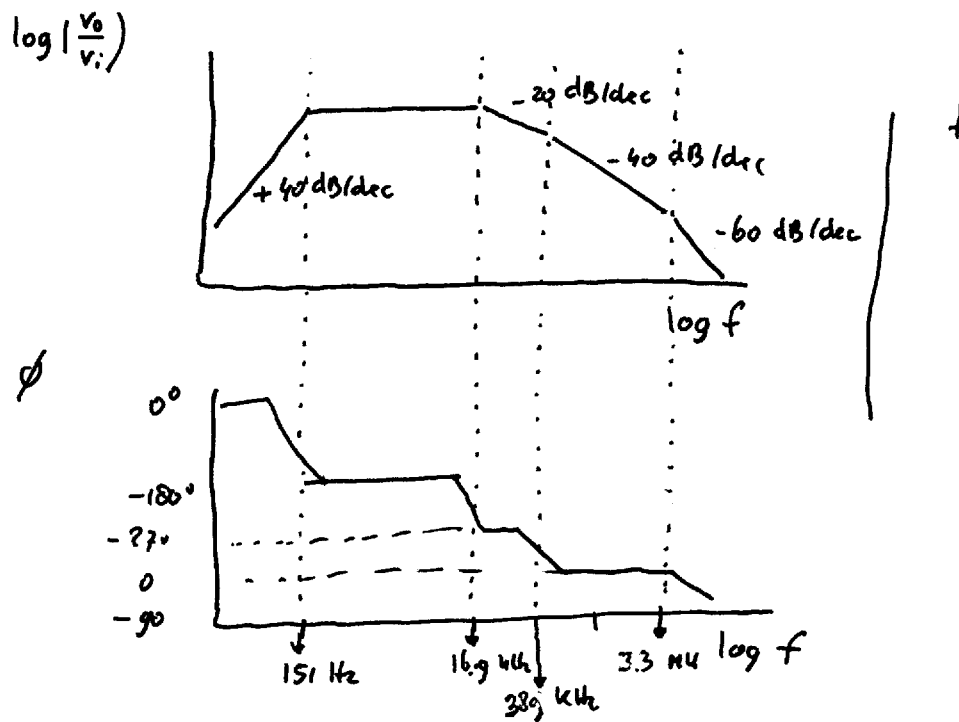
$$f_{total} = \frac{1}{2\pi \tau_{total}} = 16.1 \text{ kHz}$$

Bandwidth: 151 Hz - 16.1 kHz

NB: the mid band gain is

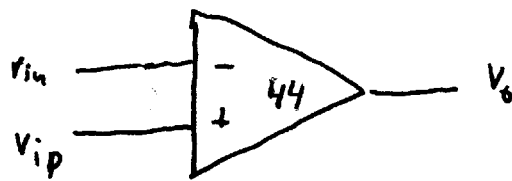
$$-44 \times \frac{r_{in}}{r_{in} + R_i} = -40$$

$$r_{in} = 2r_\pi = 10.7 \text{ k}\Omega$$



Note: the circuit was wrongly biased ($I_B = 0$ because of C_i)

Question 5



To make the circuit oscillate we have to meet the Barkhausen Criterion, $A\beta = +1$

The A of Question 2 does not have any cut-off frequencies (poles or zeroes), and we can only use

external components to have a β such that

$A\beta = +1$. We can use ^{negative} feedback with 3

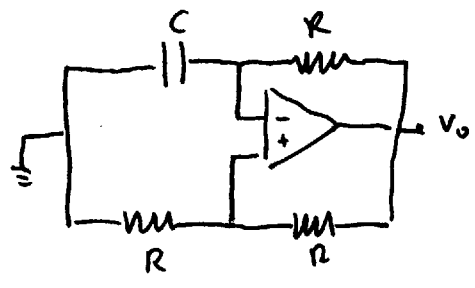
LPF filters (phase shift oscillator) or two capacitors

and an inductor (Colpitts oscillator) or two inductors

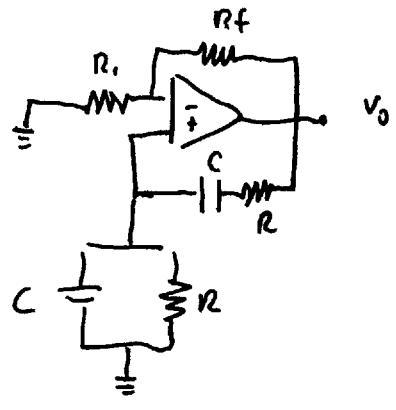
and a capacitor (Hartley oscillator) or use positive feedback

with two RC networks (Wien oscillator) or both

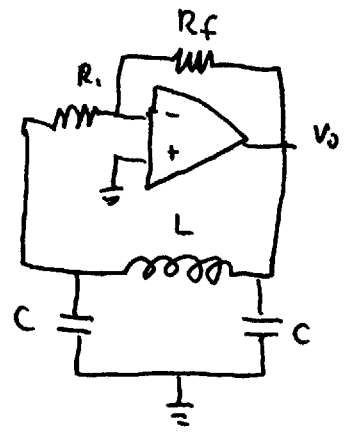
positive and negative feedback (relaxation oscillator)



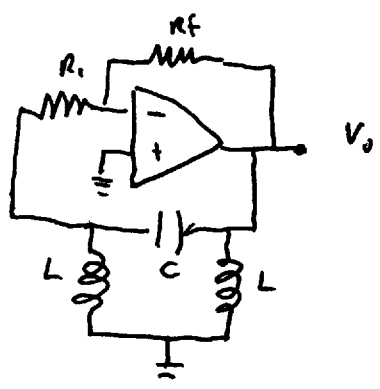
Relaxation (non Barkhausen!)



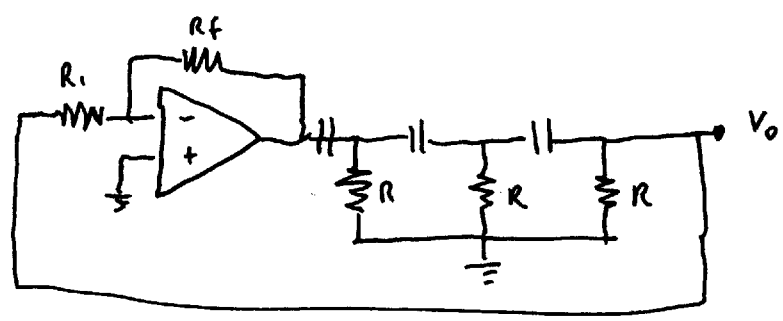
Wien



Colpitts



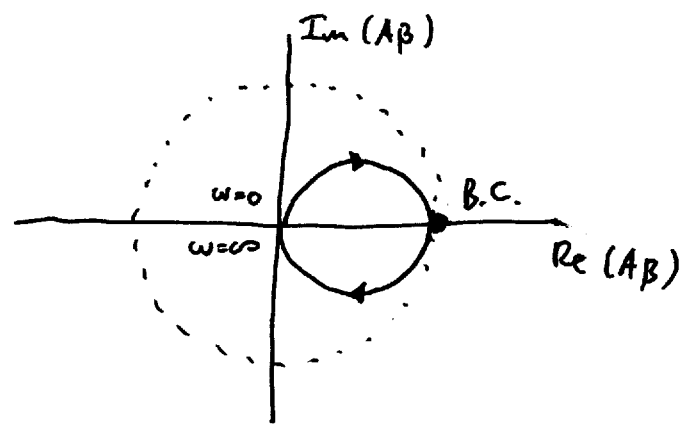
Hartley



Phase Shift

Except for the first circuit, all oscillate at the frequency where $A\beta = +1$

For instance Wien:

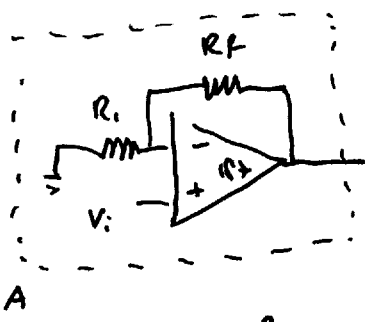


$$A\beta = A \cdot \frac{1}{3 + sRC + \frac{1}{s}RC} \quad s = j\omega$$

$$\text{Im}(A\beta) = 0 \Rightarrow j(\omega RC - \frac{1}{\omega RC}) = 0$$

$$\omega = RC$$

$$\text{Re}(A\beta) = \frac{A}{3} \stackrel{B.C.}{=} 1 \Rightarrow A = 3$$



$$A = \frac{44}{1 + 44 \times \frac{R_i}{R_i + R_f}}$$

$$\frac{R_i}{R_i + R_f} = 0.311$$

For example $R_i = 1k\Omega$
 $R_f = 2.2k\Omega$

~~///~~
 end.