

## ANÁLISE DE CIRCUITOS NO DOMÍNIO DA FREQUÊNCIA

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$$v(t) = V \sin(\omega t + \theta)$$

$$i(t) = I \sin(\omega t + \varphi)$$

$\omega$  [ $rad/s$ ] – frequência angular.

$$\omega = 2\pi f$$

$\theta$  [ $rad$ ] – fase na origem do tempo.

$f$  [ $Hz$ ] – frequência.

$$f = \frac{1}{T}$$

$T$  [ $s$ ] – período.

**Tensão eficaz de uma tensão AC:** valor de tensão DC capaz de dissipar a mesma potência média numa dada resistência.

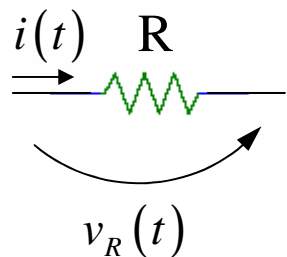
# ANÁLISE DE CIRCUITOS NO DOMÍNIO DA FREQUÊNCIA

Seja:  $i(t) = I_x \cos(\omega t)$

Definindo-se:

$$I = I_x e^{j\omega t}$$

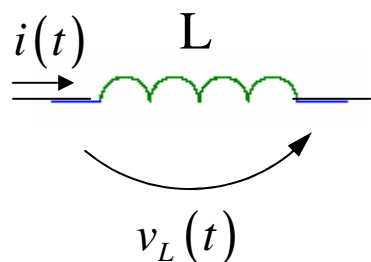
$$i(t) = \text{Re}\{I\}$$



$$V_R = RI \quad V_R = RI_x e^{j\omega t}$$

$$v_R(t) = \text{Re}\{V_R\}$$

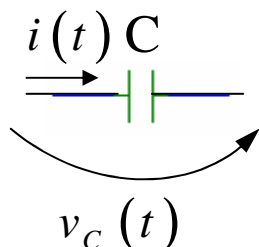
$$v_R(t) = RI_x \cos(\omega t)$$



$$V_L = Z_L I, \quad Z_L = j\omega L$$

$$v_L(t) = \text{Re}\{V_L\}$$

$$v_L(t) = \omega L I_x \cos\left(\omega t + \frac{\pi}{2}\right)$$



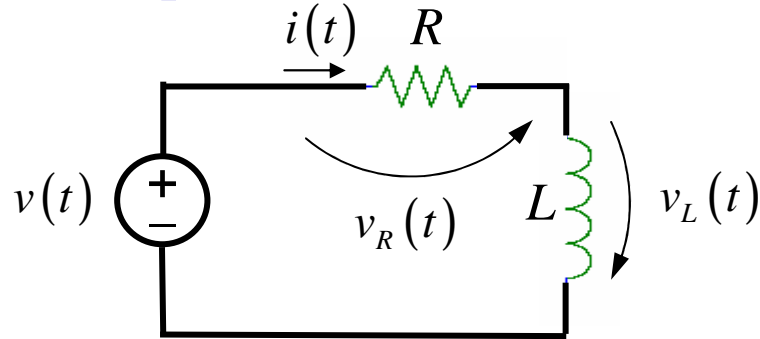
$$V_C = Z_C I, \quad Z_C = \frac{1}{j\omega C}$$

$$v_C(t) = \text{Re}\{V_C\}$$

$$v_C(t) = \frac{I_x}{\omega C} \cos\left(\omega t - \frac{\pi}{2}\right)$$

# ANÁLISE DE CIRCUITOS NO DOMÍNIO DA FREQUÊNCIA

**Exemplo:**



$$v(t) = V_s \cos(\omega t), \quad V_s = 4V, \quad \omega = 20 \text{krad/s}$$

$$R = 100\Omega \quad L = 2mH$$

$$i(t)?$$

**Lei de Kirchhoff do equilíbrio das tensões:**

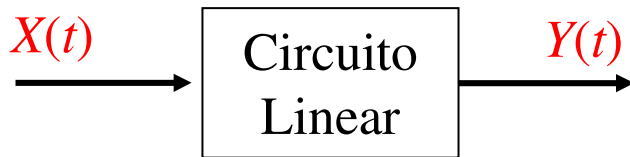
$$v(t) = v_R(t) + v_L(t)$$

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**Seja:**  $i(t) = I_s \cos(\omega t + \Phi)$

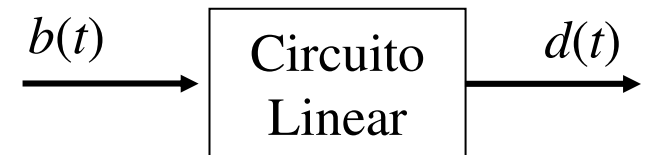
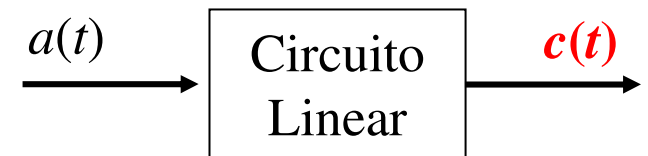
$$I_s = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}} = 37mA \quad \Phi = -\arctan \frac{\omega L}{R} = -0.38rad$$

## PRINCÍPIO DA LINEARIDADE



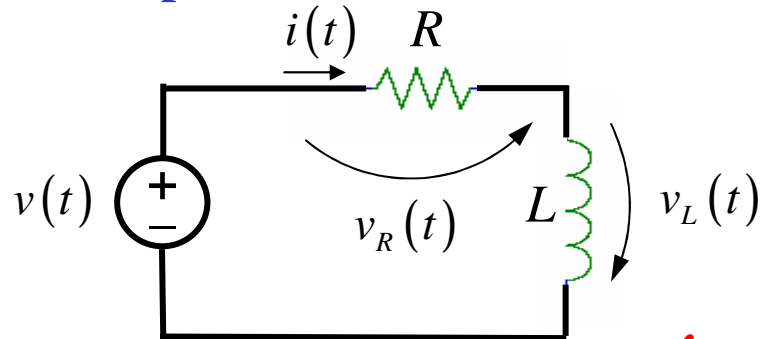
**Se**  $X(t) = a(t) + b(t)j$ ,  $Y(t) = c(t) + d(t)j$  **então:**

$$c(t) = \text{Re}\{Y(t)\}$$



# ANÁLISE DE CIRCUITOS NO DOMÍNIO DA FREQUÊNCIA

**Exemplo:**

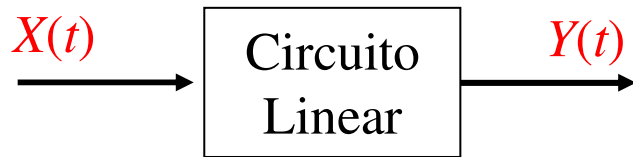


$$v(t) = V_s \cos(\omega t), \quad V_s = 4V, \quad \omega = 20 \text{ krad/s}$$

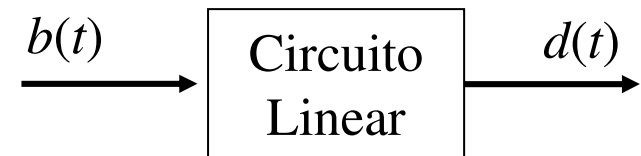
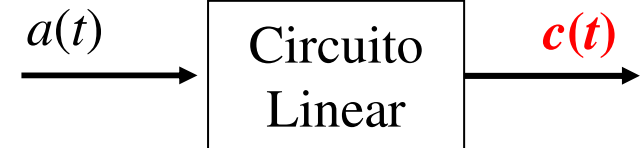
$$R = 100\Omega \quad L = 2 \text{ mH}$$

$$i(t) ?$$

## PRINCÍPIO DA LINEARIDADE



Se  $X(t) = a(t) + b(t)j$ ,  $Y(t) = c(t) + d(t)j$  então:



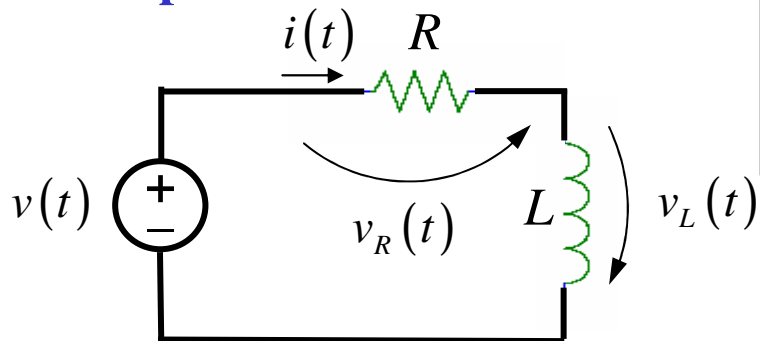
$$c(t) = \text{Re}\{Y(t)\}$$

Reais  $\longleftrightarrow$  Funções trigonométricas

Complexos  $\longleftrightarrow$  Funções exponenciais

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$$R = 100\Omega \quad L = 2 \text{ mH}$$

$$i(t)?$$

$$V = V_s e^{j(\omega t)}$$

**Seja:**  $i(t) = I_s \cos(\omega t + \Phi)$

$$I = I_s e^{j(\omega t + \Phi)}$$

**Lei de Kirchhoff do equilíbrio das tensões**

$$V = V_R + V_L$$

$$V_s e^{j\omega t} = R I_s e^{j(\omega t + \Phi)} + Z_L I_s e^{j(\omega t + \Phi)}, \quad Z_L = j\omega L$$

$$V_s e^{j\omega t} = I_s \sqrt{R^2 + \omega^2 L^2} e^{j\left(\omega t + \Phi + \arctan \frac{\omega L}{R}\right)}$$

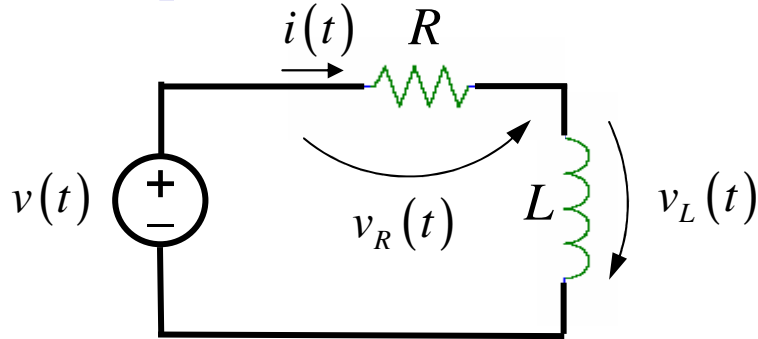
$$I_s = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Phi = -\arctan \frac{\omega L}{R}$$

$$i(t) = \text{Re}\{I\} = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \arctan \frac{\omega L}{R}\right)$$

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**Exemplo:**



$$v(t) = V_s \cos(\omega t), \quad V_s = 4V, \quad \omega = 20 \text{krad/s}$$

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$i(t)?$

**Seja:**  $i(t) = I_s \cos(\omega t + \Phi)$

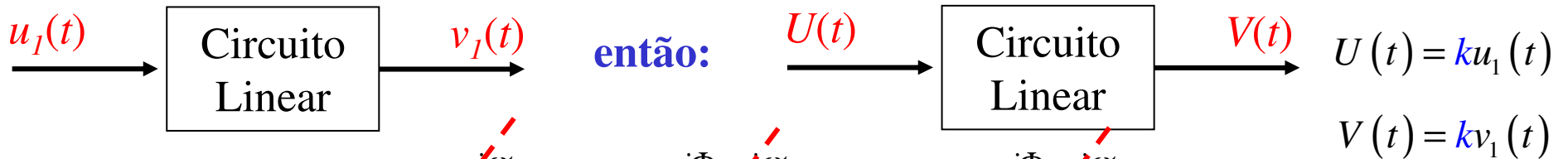
$$V = V_s e^{j(\omega t)}$$

**Lei de Kirchhoff do equilíbrio das tensões**  $V = V_R + V_L$

$$I = I_s e^{j(\omega t + \Phi)}$$

$$V_s e^{j\omega t} = R I_s e^{j(\omega t + \Phi)} + Z_L I_s e^{j(\omega t + \Phi)}, \quad Z_L = j\omega L$$

## PRINCÍPIO DA LINEARIDADE



$$V = V_s e^{j(\omega t)}$$

$$V_s e^{j\omega t} = R I_s e^{j\Phi} e^{j\omega t} + j\omega L I_s e^{j\Phi} e^{j\omega t}$$

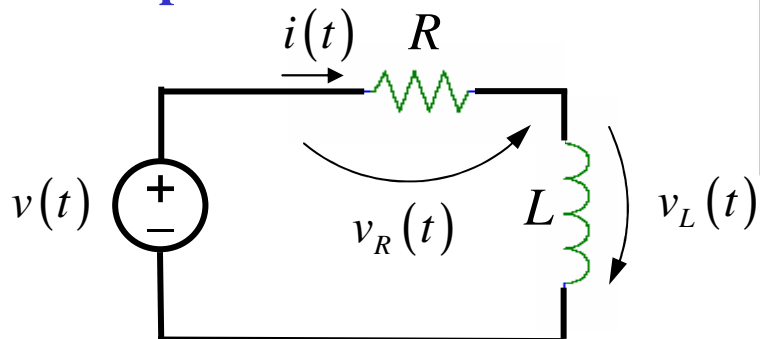
$$k \Leftrightarrow e^{j(\omega t)}$$

...

$$V_s = I_s \sqrt{R^2 + \omega^2 L^2} e^{j\left(\Phi + \arctan \frac{\omega L}{R}\right)}$$

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**Exemplo:**



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$$R = 100\Omega \quad L = 2mH$$

$$i(t)?$$

**Seja:**  $i(t) = I_s \cos(\omega t + \Phi)$

$$V = V_s e^{j(0)}$$

$$I = I_s e^{j\Phi}$$

$$V_s = I_s \sqrt{R^2 + \omega^2 L^2} e^{j\left(\Phi + \arctan \frac{\omega L}{R}\right)}$$

$$I_s = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}}$$

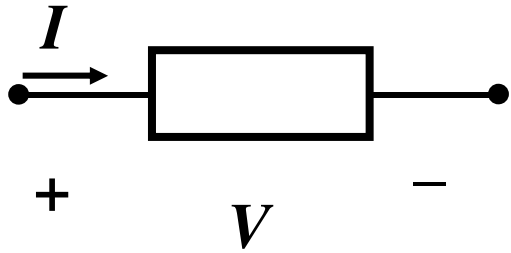
$$\Phi = -\arctan \frac{\omega L}{R}$$

$$i(t) = \text{Re}\{I e^{j\omega t}\} = \frac{V_s}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \arctan \frac{\omega L}{R}\right)$$

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## IMPEDÂNCIA GENERALIZADA



$$\frac{V}{I} = Z$$

- $Z = R$       resistência

- $Z = \frac{1}{j\omega C}$       condensador

- $Z = j\omega L$       bobine

- **SÉRIE:**  $Z_{eq} = \sum_{k=1}^n Z_k$

- **PARALELO:**  $Y_{eq} = \sum_{k=1}^n Y_k, \quad Y = \frac{1}{Z}$